Tracking of Maneuvering Target in Glint Noise Environment

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Abstract – This paper presents a tracking algorithm for maneuvering target in the presence of glint noise. In radar target tracking system, because of the random wandering of the target position, the measurement noise is clearly non-Gaussian distribution, called as glint noise, which have a considerable influence on conventional linear estimates. In this paper, the glint noise is modeled via the mixture of Gaussian distribution and Laplace distribution, and tackled with two model sets. The tracking algorithm for maneuvering target is derived detailedly when the model sets is interacting in the presence of glint noise. The Monte Carlo simulation results express its better performance in comparison with the IMM algorithm.

Keywords: Glint noise, Laplace noise, maneuvering target tracking, multiple-model

1 Introduction

In real radar tracking system, the measured position of a target is disturbed heavily due to interference of reflections from different elements of the target. This disturbed noise is the random wandering of the target position [1-3], also called as glint noise. Glint affects the measurement components (mostly the angles) by heavy tailed, non-Gaussian measurement noise, which can be characterized by probability density function (pdf) such as Student t distribution or Gaussian distribution combined with others (Lapace, etc) [4]. The non-stationary and non-Gaussian measurement noise caused by the glint severely degrades the performance of linear Kalman filter [5]. Among the known approaches when the measurement noise is non-Gaussian, the score function method presented by Masreliez [6] had an interesting feature and a relatively good performance. But this approach needs the pdfs of measurement error is known, and assumed that the pdf of state prediction is Gaussian, and then a numerical convolution operation was required when calculating the score function, which limited the practicality of this method. Therefore, the research [7] applied an orthogonal expansion to the measurement noise pdf and truncated the high ordered terms, which could avoid the convolution operation. But the calculation is also very complex, especially in computing the high ordered moments of known distributions and the saddle point. The approaches presented in literature [9-10] give some different solution to the tracking of maneuvering target in the presence of glint, and better performance is achieved.

This paper presents a tracking algorithm for maneuvering target in the presence of glint noise. In radar target tracking system, the measurement noise of glint is modeled by the mixture of Gaussian distribution and Laplace distribution, and tackled with two model sets. The tracking algorithm for maneuvering target is derived detailedly when the model sets is interacting in the presence of glint noise. The Monte Carlo simulation results express its validity in comparison with the IMM algorithm.

2 The formation of tracking maneuvering target in the presence of glint

The tracking for maneuvering target is often connected with the estimation of hybrid system, which can be described as following:

\[ x(k) = F_{k-1}(\theta_1) x(k-1) + G_{k-1}(\theta_1) w_{k-1}(\theta_1) \]  (1)

\[ z(k) = H_{\theta_1}(\theta_1) x(k) + v_{\theta_1}(\theta_1) \]  (2)

Where, \( x(k) \in \mathbb{R}^n \) is the state of maneuvering target at time \( k \), \( z(k) \in \mathbb{R}^m \) is the measurement vector of the target with additive noise. \( \theta_1 \) stands for the moving mode of the target at time \( k \), \( w_{\theta_1}(\theta_1) \) and \( v_{\theta_1}(\theta_1) \) are independent mutually, and their variances \( Q_{\theta_1} \), \( R_{\theta_1} \) are additive noise sequences respectively with zero mean. It is clear that the system is a non-linear system, and \( x(k) \), \( z(k) \) are dependent on the same uncertain moving mode of the target. If the moving mode of maneuvering target is determined, then the system model can be simplified as a linear system.

The multiple model method(MM) is a better approach to deal with the estimation of hybrid system [9-12]. In general, the MM approaches consider the measured noise...
is Gaussian sequence. If the measured noise is non-Gaussian, the performance of the estimation for the hybrid system is severely affected. Therefore, the literatures of [9-10] present different algorithm based on the MM and get better performance when tracking maneuvering target. A new algorithm is derived detailledly in this paper on the basis of interacting model sets to tackle with the glint when tracking maneuvering target.

3 A general description of glint noise

The glint noise is clearly non-Gaussian and long-tailed. One typical record about glint noise is shown in Figure 1. Lots of researches on the modeling to the glint noise have been significant effects [4,7,11]. Borden and Mumford[11] consider the distribution of it as a student’s $t$ with two degrees of freedom and develop a method to produce glint-like signals. The literature [4] argue that the glint can be modeled as a mixture of a Gaussian distribution and other noise(outlier). The fundamental of their conclusion are from the normal analysis of QQ-plot to the glint noise records. A QQ-plot is a plot of the ordered data $x_{(i)}$ versus the normal quantiles $\Phi^{-1}(p_i)$, and $\Phi^{-1}$ is the inverse of the standard normal distribution function, where

$$p_i = (i-1/2)/n, \quad i = 1,2,\ldots,n \quad (3)$$

If a normal QQ-plot is fairly linear, this indicates that the data has a normal distribution, even in the tail [4]. The analysis of normal QQ-plot to the glint noise data of Figure 1(Figure 2) shows that the glint noise is obviously non-Gaussian and has long-tailed characteristic. The heavy-tailed aspect of the marginal distribution of glint noise is related to the large glint spikes [4], which have a considerable influence on conventional linear estimates.

A general modeling to the glint noise is the mixture of Gaussian distribution and Laplace distribution. The pdf of glint noise can be described as the following:

$$f(x) = (1-\epsilon)f_g(x) + \epsilon f_l(x) \quad (4)$$

Where, $\epsilon$ is a small positive number less than one, which stands for the probability of glint appearance, and $f_g(x), f_l(x)$ represent the pdfs of Gaussian distribution and Laplace distribution respectively. When they are one dimension and zero mean, the pdfs can be given respectively as follows

$$f_g(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right) \quad (5)$$

$$f_l(x) = \frac{1}{2\eta} \exp\left(-\frac{|x|}{\eta}\right) \quad (6)$$

Where, $\sigma, \eta$ are the standard deviation of Gaussian distribution and the distributed parameters of Laplace distribution respectively.

4 Model sets interacting for the tracking of maneuvering target

Let’s begin to consider the tracking of maneuvering target when the measurement noise is glint in this paper, and the following assumptions are given as:

(1) Because of the difference of the scattering parameters of the different position of the target, and the incident angles of radar beam are also different in different instant, so the glint noise intensity are varying in different interval, which can be expressed through the varying of the measurement noise variance.

(2) In different instant, the modes for the target moving are different, which can be described via the state equations of the hybrid system adequately.

(3) The glint noise can be modeled as the mixture of Gaussian distribution and Laplace distribution.

Based on the above assumption, two filtering banks are designed, where the first set $M^1 = \{M^1_j | j = 1,2,\ldots,r\}$ stands for the moving modes of the maneuvering target where the measurement noise is Gaussian noise, and the second set $M^2 = \{M^2_j | j = 1,2,\ldots,r\}$ stands for the
moving cases where the measurement noise is Laplace noise. Simultaneously, assume that the model sets are independent mutually, and the models within the set are also independent with each other. Assuming that the jumping process between the model sets and among the different models within the set are all hemi-Markov process, and the following definitions are given as:

The measurement sequence:

\[ Z^k = \{ z_1, z_2, \ldots, z_k \} \]

The probability of the model set:

\[ \mu_{M^i} (k) = P(M^i(k)|Z^k) \quad i=1,2 \]

The probability of the model:

\[ \mu_{M^i, r} (k) = P(M^i_r(k)|Z^k) \]

\[ i=1,2 \quad r=1,2,\ldots,r \]

The transition probability between the sets:

\[ p^{i\beta} = P(M^i(k)|M^\beta(k-1)) \quad i, \beta \in \{1,2\} \]

The transition probability among the models:

\[ \rho_{mn} = P(M^m_n(k)|M^m_n(k-1)) \]

\[ i=1,2 \quad m,n \in \{1,2,\ldots,r\} \]

According to the above assumptions, there are equations definitely as follows:

\[
\sum_{i=1}^{2} \mu_{M^i} (k) = 1 \quad \sum_{j=1}^{r} \mu_{M^i, r} (k) = 1
\]

\[
\sum_{i=1}^{2} p^{i\beta} = 1 \quad \sum_{m,n=1}^{r} \rho_{mn} = 1
\]

Additionally, assuming that the switching is in the model sets or among the models within the sets, and then the following equation is clearly correct:

\[ M^1 \cap M^2 = \emptyset \]  

(7)

According to the total probability theory, the posterior pdf for the state of the maneuvering target can be shown as the following based on the effective model sets at time \( k \):

\[
p(x|Z^k) = \sum_{i=1}^{2} \mu_{M^i} (k) p(x|M^i(k),z(k),Z^{k-1})p(M^i(k)|Z^k)
\]

(8)

And the posterior pdf can also be described as:

\[
p(x|M^i(k),z(k),Z^{k-1}) = \frac{p(z(k)|M^i(k),x(k))}{p(z(k)|M^i(k),Z^{k-1})}p(x(k)|M^i(k),Z^{k-1})
\]

(9)

Assuming that the unbiased estimation for the state and its covariance at time \( k-1 \) are:

\[ \hat{x}_{M^i}(k-1) = E[x(k-1)|M^i(k-1),Z^{k-1}] \]

\[ P_{M^i}(k-1) = E[\|x(k)-\hat{x}_{M^i}(k-1)\|^2 |M^i(k-1),Z^{k-1}] \]

Then the first term of equation (8) can be rewritten as:

\[
p(x(k)|M^i(k),Z^{k-1}) = \sum_{j=1}^{r} p(x(k)|M^i(k),\hat{x}_{M^j}(k-1),P_{M^j}(k-1))p^{i\beta}(k-1)
\]

(10)

Where, \( p^{i\beta}(k-1) = P(M^i(k-1)|M^\beta(k-1),Z^{k-1}) \) stands for the probability of the set \( M^\beta \) at time \( k-1 \) conditioned on the effective model set \( M^i \) at time \( k \).

Therefore, the model sets interacting algorithm can be designed, which one cycle of the algorithm consists of the following:

1. **The initialization of the model sets**

\[
\tilde{z}^0_{M^i}(k-1) = E[x(k-1)|M^i(k),Z^{k-1}] 
\]

\[
= \sum_{j=1}^{2} \hat{x}_{M^j}(k-1)p^{i\beta}(k-1)
\]

(11)

\[
p_{M^i}^0(k-1) = E[\|\tilde{z}_{M^i}(k-1)-x(k-1)\|^2 |M^i(k),Z^{k-1}]
\]

\[ = P_{M^i}(k-1) + \sum_{j=1}^{2} \hat{x}_{M^j}(k-1)-\tilde{z}_{M^j}(k-1))][\cdots]p^{i\beta}(k-1) \]

Where, \( i,j \in \{1,2\} \)

2. **Filtering conditioned on the initialization of the model sets**

On the basis of the initialization of the model set \( M^* \) at time \( k-1 \) and the new measurement \( z(k) \), running the filtering to get the estimation at time \( k \) :

\[ \hat{x}_{M^i}(k), P_{M^i}(k) \]

and calculating the likelihood function and the conditional model probability for each model of the set:

\[
\mathcal{L}_n^i(k) = p(x(k)|M^i_n(k),\tilde{z}^0_{M^i}(k-1),P^0_{M^i}(k-1))
\]

(13)

\[
\mu_{M^i}^n(k) = P(M^i_n(k)|Z^k)
\]

\[
= \frac{p(z(k)|M^i_n(k),Z^{k-1})p(M^i_n(k)|Z^{k-1})}{p(z(k)|Z^{k-1})}
\]

(14)

\[
\mathcal{L}_n^i(k) \sum_{m,n=1}^{r} \rho_{mn} \mu_{M^m_n}^i(k-1)
\]

\[ \sum_{n=1}^{r} \mathcal{L}_n^i(k) \sum_{m,n=1}^{r} \rho_{mn} \mu_{M^m_n}^i(k-1) \]

(15)

Where, \( n,m,l \in \{1,2,\ldots,r\} \)

3. **Combination of the state and its covariance conditioned on the model sets**

\[ \hat{x}_{M^i}^k = E[x(k)|M^i(k),Z^k] \]

\[ = \sum_{n=1}^{r} \mu_{M^i_n}^n(k) \hat{x}_{M^i_n}^k \]

(16)

Where, \( i \in \{1,2\} \)
(4) The updated of the model sets

\[ \mu_{M_1}(k) = P[M_1(k) | Z^{k-1}] \]
\[ = P[z(k) | M_1(k), Z^{k-1}] \]
\[ = \frac{1}{C} \sum_{i=1} C \rho_{i} \rho_{i} \sum_{j=1} C \rho_{j} \mu_i (k-1) \]

(5) Combination of overall estimates

\[ \hat{x}[k] = E[x(k) | Z^{k}] = \sum_{i=1} \mu_{M_i}(k) \hat{x}_{M_i}(k) \]
\[ P_{M_i}(k) = E[(\hat{x}(k) - x(k)) (\hat{x}(k) - x(k))' | Z^{k}] \]
\[ = \sum_{i=1} \mu_{M_i}(k) \hat{P}_{M_i}(k) + (\hat{x}_{M_i}(k) - \hat{x}(k)) (\hat{x}_{M_i}(k) - \hat{x}(k))^T \]

The structure for the algorithm is shown as following:

Let \( \alpha \) denote the viscous drag coefficient, and \( \mathbf{x}(t)=[x(t) \ y(t) \ y(t) \ z(t) \ \dot{z}(t)]^T \), \( \mathbf{w}(t)=[w_x(t) \ w_y(t) \ w_z(t)]^T \), \( \mathbf{u}(t)=[u_x(t) \ u_y(t) \ u_z(t)]^T \) denote the state vector of the target, the driving noise and deterministic inputs respectively. Assuming that the driving noise and the inputs are piecewisely constant, then the above equation (21) can be discretized as:

\[ \mathbf{x}(k+1) = \mathbf{F} \mathbf{x}(k) + \Gamma (\mathbf{w}(k) - \mathbf{u}(k)) \]

Where, \( \mathbf{x}(k) = (x_k \ \dot{x}_k \ y_k \ \dot{y}_k \ z_k \ \dot{z}_k)^T \) stands for the state vector for the target at time \( k \), and \( \mathbf{w}(k) \) is the Gaussian noise with zero mean, \( \mathbf{u}(k) \) is the deterministic input vector, and the coefficients matrices are as follows:

\[ \mathbf{F} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \]

\[ \Gamma = \begin{bmatrix} b_1 \\ b_2 \\ 0 \\ 0 & b_1 \\ 0 \end{bmatrix} \]

Where:

\[ a_1 = (1-e^{-\alpha t})/\alpha \]

\[ a_2 = a_1 \]

\[ b_1 = (e^{-\alpha t} - 1 + \alpha t)/\alpha^2 \]

\[ b_2 = b_1 \]

Where the notation \( t \) is the sampling interval. Because the measurement is obtained in the spherical coordinate, the measurement equation can be discretized as the following:

\[ \mathbf{z}(k) = \begin{bmatrix} r(k) \\ b(k) \\ e(k) \end{bmatrix} = \begin{bmatrix} \sqrt{x_k^2 + y_k^2 + z_k^2} \\ \arctan \frac{y_k}{x_k} \\ \arctan \frac{z_k}{\sqrt{x_k^2 + y_k^2}} \end{bmatrix} + \mathbf{v}(k) \]

Where, \( \mathbf{v}(k) = (v_r \ v_b \ v_e) \) is the measurement noise, which is non-Gaussian but glint noise, whose pdf can be described via the equation (4). Additionally, \( r, b, e \) represent the range, elevation and bearing measurements respectively. It is clear that the above equation is nonlinear. In order to apply the extended Kalman filter, expand the nonlinear function in the observation equation (28) using the Taylor series and truncate the high-order terms, then we can obtain a linear observation equation:

\[ \mathbf{z}(k) = \mathbf{H}_k \mathbf{x}(k) + \mathbf{v}(k) \]

Where, \( \mathbf{H}_k \) is the coefficient matrix, which can be described as
\[
H_k = \begin{pmatrix}
\frac{s_k}{r_k} & 0 & \frac{z_k}{r_k} & 0 \\
-\frac{s_k}{(s_k^2 + z_k^2)} & 0 & \frac{z_k}{(s_k^2 + z_k^2)} & 0 \\
-\frac{s_k}{(s_k^2 + z_k^2)^{0.5}} & 0 & \frac{z_k}{(s_k^2 + z_k^2)^{0.5}} & 0 \\
-\frac{s_k}{(s_k^2 + z_k^2)^{0.5} + r_k^2} & 0 & \frac{z_k}{(s_k^2 + z_k^2)^{0.5} + r_k^2} & 0 \\
\end{pmatrix}
\]  

(30)

5.2 The design for the tracking scenario and the analysis for the Monte Carlo simulation

Consider the target moving according to the equation (31), which can be illustrated as Figure 4.

\[
U = \begin{cases}
-100m/s^2 & 20m/s^3 & -10m/s^2 \\
-150m/s^2 & 40m/s^3 & 20m/s^2 \\
-100m/s^2 & 20m/s^3 & -10m/s^2 \\
\end{cases}
\quad 0 \leq k < 30
\]

\[
U = \begin{cases}
-100m/s^2 & 40m/s^3 & 20m/s^2 \\
\quad 30 \leq k < 40
\end{cases}
\]

\[
U = \begin{cases}
-100m/s^2 & 20m/s^3 & -10m/s^2 \\
\quad 40 \leq k \leq 50
\end{cases}
\]

The parameters of the dynamic system are as follows:

\[ \varepsilon = 0.05, \quad t = 0.5s, \quad \alpha = 0.5/s \]

The initial state of the target and its initial covariance are as following respectively:

\[ x(0) = \begin{bmatrix} 10000 & -0.3 & 1 & 0.06 & 0.03 \\ 2 & 2 & 2 & 2 & 2 \end{bmatrix}, \quad Q = \text{diag}(40^2/s^4, 10^2/s^4, 5^2/s^4) \]

The measurement noise of Gaussian distribution for the hybrid system are divided into two situations, which can be represented as:

\[
R_1 = \text{diag}(50^2, 0.04, 0.4) \]

\[
R_2 = \text{diag}(100^2, 0.8, 0.8) \]

And the parameters for the Laplace noise can also be described as the following:

\[
R_1 = \text{diag}(800^2, 18, 18) \]

\[
R_2 = \text{diag}(1800^2, 50, 50) \]

The transition probability matrix is \( P_{\text{trans}} \), and the transition probability among the models within the model sets \( P_{\text{trans1}} \) and \( P_{\text{trans2}} \), respectively.

\[
P_{\text{trans}} = \begin{bmatrix} 0.95 \ 0.05 \\ 0.95 \ 0.05 \end{bmatrix}
\]

\[
P_{\text{trans1}} = P_{\text{trans2}} = \begin{bmatrix} 0.8 & 0.15 & 0.025 & 0.025 \\ 0.15 & 0.8 & 0.025 & 0.025 \\ 0.05 & 0.05 & 0.8 & 0.1 \\ 0.05 & 0.05 & 0.1 & 0.8 \end{bmatrix}
\]

The tracking results for the above maneuvering scenario using IMM algorithm and the proposed algorithm are illustrated in Figure 5 and Figure 6, which are the RMSE of the estimation for the range and velocity respectively via 200 runs of Monte Carlo simulation. From the simulation results, a conclusion can be drawn that the performance of the proposed algorithm is better than that of the IMM algorithm when the measurement noise is glint noise.

6 Conclusion

A new algorithm is presented based on the interacting for the model sets in the presence of glint noise. The glint noise is modeled with the mixture of the Gaussian distribution and Laplace distribution, and tackled with two model sets. The tracking algorithm for manoeuvring target is derived detailedly when the model sets is switching in the presence of glint noise. The Monte Carlo simulation results express its validity in comparison with the IMM algorithm.
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