Expected-Mode Augmentation for Multiple-Model Estimation

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Abstract – In this paper a new approach, referred to as expected-mode augmentation (EMA), for multiple-model (MM) estimation is proposed. It is intended to enhance the performance of an MM algorithm in the cases of a continuous mode space. In this approach, the original fixed or variable model set is augmented by a model that intends to match the expected value of the true mode, which is readily computed from the MM estimator as a probabilistically weighted sum of modal states over the model set. This makes it possible to cover at a certain accuracy level a large mode space by a relatively small number of models. Theoretical analysis and justification of the approach is presented. Performance evaluation is conducted by simulation of several EMA-IMM designs for maneuvering target tracking.

Keywords: Adaptive Estimation, Multiple Model, IMM, Variable Structure, Target Tracking

1 Introduction

In many applications of MM estimation, the set of all possible values of the uncertain system parameter, known as mode space, is continuous. As shown in this paper, optimal use of more models in the continuous mode space does improve the performance of an MM algorithm for base state estimation. In practice, however, only a limited number of models can be used. The common practice in MM estimation is to define a finite set of models to approximate this mode space. Loosely speaking, the major objective here is to achieve maximum modelling accuracy at a minimum number of models. It is mainly here that the variable-structure approach (see e.g., [8, 6, 5]) to MM estimation has certain advantages. In a general setting, the problem of efficient (desirably optimal) model set design for MM estimation (both fixed- and variable-structure (VS)) is still open.

It is known that the performance of an MM configuration highly depends on the distance of the true mode from the MM set. The shorter this distance is, the better. Thus it is highly desirable that a designed MM set has at any time at least one model which is as close to the true mode as possible. Since the true mode is time varying over a large space, if a fixed set of models is used, the required number of models to achieve satisfactory accuracy will be prohibitively large. However, nothing really prevents us from using a time-varying set of models. To capture various possible mode jumps and in the meantime to have at least one model close to the true mode, a natural idea is to use a fixed set of models, augmented by a model that follows closely the true mode. A good candidate for the augmenting model is the posterior mean of the mode since it is statistically closest to the true mode. Furthermore, this expected mode is readily available from an MM algorithm as sum of modal states weighted by the corresponding posterior model probabilities. This approach is systematic and general for all problems with a continuous mode space. It is therefore proposed and investigated in this paper.

Several researchers have considered similar problems and proposed their solution techniques. The use of an initial coarse grid and a subsequent fine grid was proposed in [2] for a static MM algorithm. Also for a static MM algorithm, [12] presented a filter bank that moves over a predefined fixed grid according to a decision logic. It was proposed in [13, 14] to use a moving set of acceleration models centered around a model whose acceleration is determined by an additional Kalman filter. In [3], an adaptive IMM algorithm for radar tracking of a maneuvering target was proposed that uses an acceleration model determined by a separate Kalman filter on top of a fixed set of models. In [8], it was suggested to employ the expected mode as center of an adaptive grid for an example of non-stationary noise identification.

In the approach proposed in this paper, the original fixed or variable model-set is augmented by the expected mode.
computed as a probabilistically weighted sum of modal states by the MM algorithm. Compared with the existing techniques described above, it is not only much more general and systematic, but also highly cost-effective and easy to implement.

2 Expected-Mode Augmentation Approach

In this section, we explain and justify the proposed EMA approach.

2.1 Convex Combination of Estimates

Denote by \( \hat{x}_1 \) and \( \hat{x}_2 \) any two distinct unbiased estimators of \( x \). Define a new estimator as a convex combination of \( \hat{x}_1 \) and \( \hat{x}_2 \):

\[
\hat{x} = \alpha_1 \hat{x}_1 + \alpha_2 \hat{x}_2
\]

where \( \alpha_1 + \alpha_2 = 1, 0 < \alpha_i < 1 \). We would like to determine the condition under which \( \hat{x} \) is better than \( \hat{x}_1 \). From

\[
\hat{x} = x - \hat{x} = \alpha_1 \hat{x}_1 + \alpha_2 \hat{x}_2, \quad \hat{x}_1 = x - \hat{x}_1
\]

it follows that the mean-square error (mse) of \( \hat{x} \) is

\[
\mathcal{E} = E[\hat{x}^2] = E[(\alpha_1 \hat{x}_1 + \alpha_2 \hat{x}_2)^2]\]

\[
= \alpha_1^2 E[\hat{x}_1^2] + \alpha_2^2 E[\hat{x}_2^2] + 2 \alpha_1 \alpha_2 E[\hat{x}_1 \hat{x}_2]
\]

Since \( \hat{x}_1 \) and \( \hat{x}_2 \) are unbiased, if their estimation errors are uncorrelated (i.e., orthogonal), we have

\[
\mathcal{E} = \alpha_1^2 \mathcal{E}_1 + \alpha_2^2 \mathcal{E}_2
\]

where \( \mathcal{E}_i \) is the mse of \( \hat{x}_i \). Since \( \alpha_1 = 1 - \alpha_2 \), we have

\[
\mathcal{E} < \mathcal{E}_1 \text{ if and only if } (1 - 2 \alpha_2 + \alpha_2^2) \mathcal{E}_1 + \alpha_2^2 \mathcal{E}_2 < \mathcal{E}_1
\]

or equivalently

\[
\mathcal{E}_2 < \frac{2 - \alpha_2}{\alpha_2} \mathcal{E}_1
\]

That is, the convex combination \( \hat{x} \) is better than \( \hat{x}_1 \) if and only if the mse\((\hat{x}_2) \) is less than \( \frac{2 \mathcal{E}_1}{\mathcal{E}_1 + \mathcal{E}_2} \) or equivalently

\[
\alpha_2 < \frac{2 \mathcal{E}_1}{\mathcal{E}_1 + \mathcal{E}_2}
\]

This inequality is always satisfied if \( \mathcal{E}_2 < \mathcal{E}_1 \), that is, if \( \hat{x}_2 \) is better than \( \hat{x}_1 \), as illustrated in Figure 1(a). Even if \( \hat{x}_2 \) is worse than \( \hat{x}_1 \), this inequality shows that \( \hat{x} \) is still better than \( \hat{x}_1 \) provided \( \alpha_2 \) satisfies the above inequality, as illustrated in Figure 1(b). In Figure 1, the distance measure is the standard error (square-root of mse)

\[
\|\hat{x}\| = (E[\hat{x}^2])^{1/2} = (\mathcal{E})^{1/2},
\]

the line that connects \( \hat{x}_1 \) and \( \hat{x}_2 \) represents all possible points of \( \hat{x} \) and its solid line portion represents the points at which \( \hat{x} \) is better than \( \hat{x}_1 \).

The above result relies on the assumption that \( \hat{x}_1 \) and \( \hat{x}_2 \) are orthogonal. As illustrated in Figures 1(c) and 1(d), if they are not orthogonal, it is possible for \( \hat{x} \) to be better than \( \hat{x}_1 \) (i.e., \( \hat{x} \) with the optimal \( \alpha_1 \) and \( \alpha_2 \) is better than \( \hat{x}_1 \)) if and only if

\[
E[\hat{x}_1^2] < E[\hat{x}_1^2]
\]

that is, the projection of \( \hat{x}_2 \) on \( \hat{x}_1 \) either has a smaller magnitude than \( \hat{x}_1 \), if they are in the same direction, or has a direction that is opposite to \( \hat{x}_1 \). In other words, use of \( \hat{x}_2 \) may still be beneficial even if \( \hat{x}_2 \) is worse than \( \hat{x}_1 \) and their estimation errors are correlated. In fact, it can be easily shown that the optimal \( \alpha_2 = \frac{\mathcal{E}_1}{\mathcal{E}_1 + \mathcal{E}_2} \), which is the bisecting point of the solid line portion (Figure 1) at which \( \hat{x} \) is orthogonal to \( \hat{x}_2 = \hat{x}_1 \).

2.2 Benefit of Model-Set Augmentation

We now apply the above general result to MM estimation. Denote by \( s \) the true mode and by \( S \) the mode space (i.e., the set of possible values of \( s \)). Consider the problem of adding a model set \( C \) to the original model set \( M \), (hence \( C \cap M = \emptyset \)). Assume \( (M \cup C) \subset S \). For simplicity of presentation, assume the problem is a static one (i.e., time-invariant), although the same idea works just fine for time-varying cases. Let

\[
\hat{x}_M = E[x | s \in M, z]
\]

\[
\hat{x}_C = E[x | s \in C, z]
\]

\[
\mu_M = P\{s \in M | s \in (M \cup C), z\}
\]

\[
\mu_C = P\{s \in C | s \in (M \cup C), z\}
\]

where \( z \) stands for measurement. Then the estimator of \( x \) based on the union of model sets \( M \) and \( C \) is
\[ \begin{align*}
\hat{x} &= E[x | s \in (M \cup C), z] \\
&= E[x | s \in M, z]P\{s \in M | s \in (M \cup C), z\} \\
&\quad + E[x | s \in C, z]P\{s \in C | s \in (M \cup C), z\} \\
&= \mu_M \hat{x}_M + \mu_C \hat{x}_C 
\end{align*} \]

which is a convex combination of \( \hat{x}_M \) and \( \hat{x}_C \). The above result is therefore applicable if \( \hat{x}_M \) and \( \hat{x}_C \) are unbiased (and have uncorrelated estimation errors), which can be assumed in most cases. That is, use of the union of \( M \) and \( C \) is better than use \( M \) alone if and only if

\[ \mu_C < \frac{2\xi_M}{\xi_M + \xi_C} \]

This inequality is always satisfied if \( \xi_C < \xi_M \), that is, if \( \hat{x}_C \) is better than \( \hat{x}_M \). Even if \( \hat{x}_C \) is worse than \( \hat{x}_M \), \( \hat{x} \) is still better than \( \hat{x}_M \) provided \( \mu_C \) satisfies the above inequality, and in the case where \( E[\hat{x}'_M \hat{x}_C] \neq 0 \), \( \hat{x} \) may still be better than \( \hat{x}_M \) if

\[ E[\hat{x}'_M \hat{x}_C] < E[\hat{x}'_M \hat{x}_M] \]

Note that this result, which holds when \( M \cup C \subset S \), does not contradict the finding presented in [8] that the optimal use of more models is not necessarily better because the above result would not necessarily be correct if \( C \subset S \) were not true. Since we are focusing in this paper on problems with a continuous mode space, \( M \cup C \subset S \) holds in general and thus

\[ \hat{x}_a = (1 - \alpha)\hat{x}_M + \alpha \hat{x}_C \]

with some \( \alpha < \frac{2\xi_M}{\xi_M + \xi_C} \) will be better than \( \hat{x}_M \). As a consequence, optimal use of more models for such problems does improve performance because its estimate \( \hat{x} \) cannot be worse than \( \hat{x}_a \). Note, however, that this holds only under the assumption that \( s \in (M \cup C) \), which is not true in general.

The above results have many potential applications in model set design for fixed-structure MM estimation as well as variable-structure MM (VSMM) estimation. We consider only one application here in adaptive-grid VSMM estimation. We can use at every time the union of two model sets \( M \) and \( C \), where \( M \) is a set of fixed models while \( C \) is a set of adaptive models. As illustrated in Figure 2, they represent a fixed coarse grid and an adaptive fine grid, respectively. A VSMM algorithm based on this idea can be called a "hybrid-grid" MM estimator, where hybrid grid represents both fixed/adaptive and coarse/fine grids. How should the set \( C \) change is a separate issue to be addressed in a future paper. Figure 2 illustrates a moving grid—one of many possible schemes for grid adaptation.

### 2.3 Expected-Mode Augmentation

Now suppose that from computational consideration it is desirable to add only one model to the set \( M \), that is, \( C = \{m_a\} \), as illustrated in Figure 3. What is the best model \( m_a \) to use based on all information available at time \( k \) in the MM estimator in the sense of having the best chance of improving on \( \hat{x}_M \)? Since the mode space \( S \) is continuous, one candidate is clearly the expected mode, that is, the conditional mean of \( s(k) \) given data and model set \( M_k \). It is certainly the best model \( m \) that is closest to the true mode \( s \in S \) with the distance measure

\[ ||s - m|| = (E[(s - m)'(s - m)] | s \in M_k, M^{k-1}, x^k)^{1/2} \]

It can be expected that

\[ \hat{x}_a = E[x(k) | s(k) = \hat{s}(k), x^k] \]

is close to the optimal single-model-based estimate. There are two additional advantages of using this model: it is general—it is valid for all problems where the above \( \hat{s} \) is meaningful—and \( \hat{s} \) is readily available from an MM estimator with little extra computation.

We are now ready to formally define the expected-mode augmentation approach.

**Definition 1** (EMA set). The union of a model set \( M \) and the model that matches the above expected mode (i.e., \( m_a = \hat{s}(k) \)) is called the expected-mode augmented (EMA) set of \( M \).

**Definition 2** (EMA algorithm). An MM estimation algorithm using an EMA set is called an EMA algorithm.

The use of the word "augmentation" is justified by the following. A model set \( M \) with \( n \) models can be represented by the vector \( m = [m_1, \ldots, m_n]' \), where \( m_i \) is the modal state to which the \( i \)th model matches. The vector that represents an EMA set of \( M \) is \( m_A = [m_1, \ldots, m_n, \hat{s}]' \); that is, \( m \) augmented by \( \hat{s} \). This is similar to the state augmentation for the base state.
We now describe briefly basics of the practical EMA algorithm. Various issues involved in the actual implementation are covered in the subsequent sections. We assume for simplicity of presentation the IMM mechanism is used for model-conditioned reinitialization, [4].

Consider a generic cycle from time $k-1$ to $k$. Suppose that the model sets of a (variable-structure) MM algorithm without using EMA are $M_{k-1} = M_1$ at time $k-1$ and $M_k = M_2$ at time $k$. The corresponding EMA MM algorithm uses $M_{k-1} = M_1^+$ and $M_k = M_2^+$ respectively, where $M_j^+ = \{ M_j, \bar{s} \}$ denotes the EMA set of $M_j$, $j = 1, 2$.

A simple scheme is to compute expected mode

$$
\bar{s}(k) = \sum_{m_i \in M_1^+} m_i \mu_i(k-1)
$$

based on the model probabilities $\mu_i(k-1)$ at the previous time, or based on the current time predicted model probabilities $\mu_i(k|k-1)^1$

$$
\bar{s}(k) = \sum_{m_j \in M_2^+} m_j \mu_j(k|k-1)
$$

with $M_2^+ = \{ M_2, \bar{s}(k-1) \}$, and then run a generic VSMM cycle, such as the VSIMM($M_1^+, M_2^+$) cycle of [5] with $M_2^+ = \{ M_2, \bar{s}(k) \}$ (see [7] and [8]). The subsequent sections contain detailed implementation of this scheme.

Some other schemes, based on variable structure estimation techniques, are also possible — they will be addressed in a forthcoming article.

3 EMA-IMM Algorithms for Maneuvering Target Tracking

3.1 Tracking Problem

The target-measurement model is

$$
x(k+1) = Fx(k) + G [a(k) + w(k)]
$$

$$
z(k+1) = H x(k+1) + v(k+1), \quad k = 0, 1, 2, \ldots
$$

where $x \triangleq (x, v_x, y, v_y)'$ denotes the target state, $a \triangleq (a_x, a_y)'$ is the acceleration, $w \sim \mathcal{N}[0, Q]$ is the acceleration process noise, $z \sim \mathcal{N}[0, R]$ is the random measurement error, and

$$
F \triangleq\begin{bmatrix}
1 & T & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & T \\
0 & 0 & 0 & 1
\end{bmatrix} \quad
G \triangleq\begin{bmatrix}
T^2/2 & 0 \\
T & 0 \\
0 & T^2/2 \\
0 & T
\end{bmatrix} \\
H \triangleq\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
$$

The unknown true acceleration $a(k)$ is assumed piecewise constant, varying over a given continuous planar region $\mathcal{A}_c^c$. In the MM framework, we consider a generic finite set (grid) of acceleration values:

$$
A_r \triangleq \{ a_i \in \mathcal{A}_c^c : i = 1, 2, \ldots, r \}
$$

which defines the total model set. We approximate the evolution of the true acceleration over the quantized set $A_r$ via a Markov chain model, that is, $a(k) \in A_r$ with given $P \{ a(0) = a_i \} = P_i$ and $P \{ a(k) = a_j | a(k-1) = a_i \} = \pi_{ij}$ for $i, j = 1, 2, \ldots, r$.

3.2 Four MM Designs

Consider the following well-known example of [1], [7], [11], [14], [8], [10], [9]. The mode space is defined as

$$
\mathcal{A}_c^c \triangleq \{ (a_x, a_y) : |a_x| + |a_y| \leq 4\}
$$

i.e. the maximum acceleration in any coordinate direction is about $4g$. It is however more appropriate that the target acceleration model be independent of the orientation of the observer’s coordinate system. That is why we consider also

$$
\mathcal{A}_0^c \triangleq \{ (a_x, a_y) : \sqrt{a_x^2 + a_y^2} \leq 4\}
$$

IMM13 The basic 13-model set design $A_{13}$, obtained after quantization of $\mathcal{A}_c^c$ is

$$
\begin{align*}
\{a_1 = \rho[0, 0]' & a_2 = \rho[1, 0]' & a_3 = \rho[0, 1]' \\
\{a_4 = \rho[-1, 0]' & a_5 = \rho[0, -1]' & a_6 = \rho[1, 1]' \\
\{a_7 = \rho[-1, 1]' & a_8 = \rho[-1, -1]' & a_9 = \rho[1, -1]' \\
\{a_{10} = \rho[2, 0]' & a_{11} = \rho[0, 2]' & a_{12} = \rho[-2, 0]' \\
\{a_{13} = \rho[0, -2]'
\end{align*}
$$

with $\rho = 20 \approx 2g$.

The transition relations among models are easily understood in terms of the directed graph (i.e., digraph) representation of an MM, introduced in [8]. The topology of model set $A_{13}$ is depicted in Figure 4. Each model is viewed as a point in the mode (acceleration) space. An arrow from one model to another indicates a legitimate model switch (self-loops are omitted) with nonzero probability. All details can be found in [8], [10]. Note that for simplicity in $A_{13}$ a model is allowed to switch to its nearest neighbor(s) only. Better results could be obtained if other types of model switching are allowed, such as those between second nearest neighbors (e.g., $a_2$ and $a_3$, and $a_6$ and $a_9$) (see [9]). The values of the transition probability matrix, used in our implementation are the same as given by (7) of [10].

EMA13+1 As illustrated in Figure 5, the expected-mode augmented (EMA) set of the fixed-grid model $A_{13}$ is $A_{13+1} (k) \triangleq \{ A_{13}, \tilde{a}(k|k-1) \}$, where

$$
\tilde{a}(k|k-1) = \sum_{a_j \in A_{13+1}(k-1)} \mu_j(k|k-1) a_j
$$

$^1$They are given in a VSMM algorithm by $\mu_j(k|k-1) = \sum_{m_i \in M_1^+} \pi_{ij} \mu_i(k-1), m_j \in M_2$, where $\pi_{ij}$, are the model transition probabilities.
and \( \mu_j (k|k-1) \) are the predicted model probabilities available from the IMM estimator. Note that \( \hat{a}(k|k-1) \), as a convex combination of the points of \( A_{13} \), covers the entire continuous acceleration region \( A^c \), i.e. \( \hat{a}(k|k-1) \) can be any point in \( A^c \), depending on \( \{ \mu_j (k|k-1) \} \). The values of the transition probability matrix (TPM) \( \Pi = [\pi_{ij}] \) in the simulation are chosen based on the TPM \( P = [p_{ij}] \) of \( A_{13} \) as follows:

\[
\pi_{1,14} = 0.01, \quad \pi_{i,14} = 0.05, \quad i = 2, 3, \ldots, 13 \]
\[
\pi_{jj} = p_{jj} - \pi_{j,14}, \quad \pi_{14,j} = 0.01, \quad j = 1, 2, \ldots, 13
\]

and all other elements remain unchanged (i.e. \( \pi_{ij} = p_{ij} \)).

The implementation of the EMA13+1 tracking algorithm is straightforward with \( A_{13+1} \) \( (k) \) on receipt of the new measurement \( z_k \). In fact it is a particular case of the general VSIMM\( [A(k), A(k-1)] \) recursion for Recursive Adaptive Model-Set (RAMS) estimation with zero-memory depth \([6, 5]\).

**EMA9+1** This model, illustrated in Figure 6, is obtained from EMA13+1 by deleting its internal nonzero models (vertices) \( a_2, a_3, a_4, a_5 \). Its TPM used in sim-

\[
\begin{array}{cccccccccc}
0.96 & 0.001 & 0.001 & 0.001 & 0.001 & 0.001 & 0.001 & 0.032 \\
0.01 & 0.75 & 0.0 & 0.0 & 0.01 & 0.0 & 0.0 & 0.22 \\
0.01 & 0.0 & 0.75 & 0.0 & 0.01 & 0.0 & 0.0 & 0.22 \\
0.01 & 0.0 & 0.0 & 0.75 & 0.0 & 0.01 & 0.0 & 0.22 \\
0.01 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.22 \\
0.01 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.22 \\
0.01 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.22 \\
0.01 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.22 \\
0.01 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.22 \\
0.01 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.22 \\
\end{array}
\]

**EMA7+1** Attempting to cover more efficiently the true acceleration set \( A_7^c \) defined above we came up with a (7+1)-model set design \( A_{7+1} \), illustrated in Figure 6. It is defined by the fixed model set \( A_7 \):

\[
\begin{align*}
\{ a_1 = \rho [0, 0] \}, & \quad a_2 = \rho [2, 0] \} \\
a_3 = \rho \left[ \frac{1}{\sqrt{3}} \right] \} & \quad a_4 = \rho \left[ -1, \sqrt{3} \right] \} \\
a_5 = \rho [-2, 0] \} & \quad a_6 = \rho \left[ -1, -3 \right] \} \\
a_7 = \rho \left[ 1, -\sqrt{3} \right] \} & \quad a_8 = \rho \left[ 1, \sqrt{3} \right] \}
\end{align*}
\]

with \( \rho = 20 \).

One can easily verify that the Hausdorff distance\(^3\) between the triangular grid \( A_7 \) and the true mode set \( A_9^c : d(A_7, A_9^c) \triangleq \max_{a \in A_9^c} \min_{a_i \in A_7} ||a - a_i|| = 40\sqrt{3}/3 \approx 23.09 \) while for \( A_9 \) (with two more models) this distance is not much shorter: \( d(A_9, A_9^c) = 40.1/2 = 20 \).

The following TPM was used in the simulation

\[
\begin{array}{ccccccccccc}
0.894 & 0.001 & 0.001 & 0.001 & 0.001 & 0.001 & 0.1 \\
0.05 & 0.65 & 0.05 & 0.05 & 0.05 & 0.0 & 0.0 & 0.2 \\
0.05 & 0.05 & 0.65 & 0.05 & 0.05 & 0.0 & 0.0 & 0.2 \\
0.05 & 0.05 & 0.05 & 0.55 & 0.05 & 0.05 & 0.0 & 0.2 \\
0.05 & 0.05 & 0.05 & 0.05 & 0.65 & 0.05 & 0.05 & 0.2 \\
0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.65 & 0.05 & 0.2 \\
0.001 & 0.001 & 0.001 & 0.001 & 0.001 & 0.001 & 0.993 \\
\end{array}
\]

All other parameters of the IMM algorithms implemented in the simulation are the same as given in \([10]\), e.g. \( T = 18, Q^1 = (0.003)^2 I, Q^j = (0.008)^2 I, j \neq 1, R = 1250 I \).

\(^3\)By definition \( d(X, Y) \triangleq \max_{x \in X} \min_{y \in Y} ||x - y|| \) for compact \( X, Y \subset \mathbb{R}^n \).
random scenario the acceleration vector tricks that are effective only for certain scenarios. In the
actually impossible, to design an MM estimator with subtle
details and discussions are given in [10]. In the simulation
Gaussian with mean $\mu$ and variance $\sigma^2$, the acceleration magnitude
are deliberately chosen far apart from the grid points. As
such, for the fixed structure estimator IMM13 the scenario
DS2 is more difficult than DS1.

To provide a fair (as much as possible) performance
comparison over an ensemble of maneuver trajectories the
algorithms were tested on the random scenario, developed
from detailed simulations conducted have shown that
results between IMM13 on the one hand and EMA9+1
and in particular the fixed-grid IMM13. The comparative
results between IMM13 on the one hand and EMA9+1 (EMA7+1) on the other are scenario dependent. While
IMA13 provides less biased steady-state errors in DS1,
EMA9+1 and EMA7+1 give better accuracy for almost all
jumps in DS2. This is due to the fact (already mentioned
before) that for IMM13 DS1 and DS2 represent respectiv-
ety easy and difficult scenarios, regarding the closeness
of the true acceleration to the fixed-grid values. The EMA
algorithms are less scenario-dependent. In general how-
ever the accuracy (error) differences do not seem to be
significant.

Results over 500 MC runs of the random scenario are
given in Figure 9. Clearly all EMA designs provide bet-
ter “overall accuracy”. What is somewhat surprising is
the negligible difference between EMA13+1 and the two
other EMA designs, which have quite fewer models. A
possible explanation is that the modal estimate provides a
good “coverage” of the whole continuous region of possi-le (simulated) accelerations, even when the number of
the fixed models is small (7 and 9 resp.).

The computational complexity of the algorithms’ pro-
gram implementation, evaluated in terms of relative float-
ing point operations (FLOP) ratios is summarized in Table
2.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>DS1 $a_x(k)$</th>
<th>DS1 $a_y(k)$</th>
<th>DS2 $a_x(k)$</th>
<th>DS2 $a_y(k)$</th>
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<td>0</td>
<td>0</td>
<td>0</td>
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<td>22</td>
<td>8</td>
<td>22</td>
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<td>37</td>
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<td>27</td>
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<td>56 - 80</td>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>81 - 98</td>
<td>25</td>
<td>2</td>
<td>15</td>
<td>2</td>
</tr>
<tr>
<td>99 - 119</td>
<td>-2</td>
<td>19</td>
<td>-2</td>
<td>9</td>
</tr>
<tr>
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<td>0</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
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</tr>
<tr>
<td>151 - 160</td>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 1: Deterministic Scenarios’ Parameters

4.2 Simulation Results

Results over 100 Monte Carlo runs of DS1 and DS2 are
plotted in Figures 7 and 8. It is seen first that EMA13+1 in
both scenarios (and in all other simulated scenarios as
well, not shown) outperforms the remaining algorithms,
and in particular the fixed-grid IMM13. The comparative
results between IMM13 on the one hand and EMA9+1 (EMA7+1) on the other are scenario dependent. While
IMA13 provides less biased steady-state errors in DS1,
EMA9+1 and EMA7+1 give better accuracy for almost all
jumps in DS2. This is due to the fact (already mentioned
before) that for IMM13 DS1 and DS2 represent respectiv-
ety easy and difficult scenarios, regarding the closeness
of the true acceleration to the fixed-grid values. The EMA
algorithms are less scenario-dependent. In general how-
ever the accuracy (error) differences do not seem to be
significant.

Results over 500 MC runs of the random scenario are
given in Figure 9. Clearly all EMA designs provide bet-
ter “overall accuracy”. What is somewhat surprising is
the negligible difference between EMA13+1 and the two
other EMA designs, which have quite fewer models. A
possible explanation is that the modal estimate provides a
good “coverage” of the whole continuous region of possi-le (simulated) accelerations, even when the number of
the fixed models is small (7 and 9 resp.).

The computational complexity of the algorithms’ pro-
gram implementation, evaluated in terms of relative float-
ing point operations (FLOP) ratios is summarized in Table
2.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>IMM13 FLOP</th>
<th>EMA13+1 FLOP</th>
<th>EMA9+1 FLOP</th>
<th>EMA7+1 FLOP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.138</td>
<td>0.677</td>
<td>0.488</td>
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</tr>
</tbody>
</table>

Table 2: Computational Load

5 Conclusion

A new approach — Expected-Mode Augmented MM
estimation — has been proposed and examined. The results
detailed simulations conducted have shown that
- performance of MM estimation can be enhanced at
  the cost of a reasonable increase in computation;
- computational complexity of MM estimation can be
  reduced substantially at no/or negligible loss in accu-
  racy.

The approach is generally applicable, wherever para-
metric uncertainty arises. It can in general facilitate the
design of more efficient MM estimators.
Fig. 7: Estimation Errors (DS1)

Fig. 8: Estimation Errors (DS2)
Fig. 9: Estimation Errors (Random Scenario)

References


