Abstract - In multi-sensor and multi-target tracking systems, for data fusion, signals in presence of noise as input must be sent to fusion center to be filtered, associated, combined and made final decision as output. In the chain, association is very important processing. In this paper, an efficient fuzzy logic data association approach for tracking systems is proposed. The proposed approach is developed based on the fuzzy clustering means algorithm, which differs from many other fuzzy logic data association algorithms. Performance evaluation and results using embedded Shenyang(China) simulations are reported, and a comparison with other fuzzy logic approaches based on the results described in other references is also presented. The efficiency of the new approach has been demonstrated by the fuzzy system performance evaluation.

Keywords: data fusion, fuzzy logic, multi-sensor, multi-target, simulation

1 Introduction

In multi-sensor and multi-target tracking systems, for data fusion(Fig.1), signals in presence of noise as input must be sent to fusion center to be filtered, correlated, associated, combined and made final decision as output. In the chain, correlation and association are very important processing, especially, measurements-to-tracks correlation or association \(^{[1,2,8,12,23]}\). A number of approaches have been proposed in the literature to solve this problem ranging from sub-optimal, simple, approaches to complex, optimal, approaches \(^{[10]}\). The computational cost in generating the optimal solutions to the data association problem is usually excessive when the number of targets and the number of measurements are large. Thus the optimal solutions are not computationally feasible for real-time surveillance systems. Therefore, sub-optimal, but computationally feasible, solutions are developed. Fuzzy techniques based on fuzzy logic (whose definition can be found in other references) are well suited to model decision making processes \(^{[11]}\). Although fuzzy systems have several advantages including processing capability to uncertainty, simplicity and ease in designing, they are associated with a critical problem. As the system complexity increases, it becomes difficult to determine the right set of rules and membership functions to describe the system behavior.

The application of fuzzy approaches to multisensor-multitarget tracking systems is complicated in a dense target environment. For example, if we use conventional fuzzy logic data association techniques \(^{[16]}\) to solve the problem of associating six measurements with six tracks using only two input variable (position and speed errors) and five linguistic variables (Very Low, Low, Medium, High, Very High), the required number of IF THEN rules is \((5 \times 6)^2 \approx 900\). Thus the extension of conventional fuzzy logic systems to the case of more than three or four targets increases the computational complexity. Furthermore, the solution of the conventional fuzzy logic approach to the data association problem is an approximate solution, and the accuracy depends on several factors including the number of input variables, the number of linguistic variables, the choice of membership...
function, and the accuracy of the fuzzy rules and statements.

In this paper, an efficient fuzzy logic data association approach for MSMT tracking systems is proposed. The proposed approach is developed based on a new kind of fuzzy clustering means (FCM) algorithm\(^\text{[4,5]}\). The FCM algorithm determines a partition matrix whose elements represent the degrees of membership of data in fuzzy clusters. This approach differs from many traditional fuzzy clusters. This approach differs from many fuzzy logic data association algorithms\(^\text{[15,16]}\) which consist of four basic elements: I) fuzzification of crisp data into fuzzy variables. II) fuzzy knowledge base containing IF THEN rules and fuzzy statements. III) fuzzy inference which emulates human decision making processes to generate output fuzzy variables, and IV) defuzzification of fuzzy variable into non-fuzzy variable (crisp data). The proposed approach performs data association based on the partition matrix of data (measurement) in fuzzy clusters (tracks). The main advantages of the proposed algorithm are simplicity (it can easily be extended to a dense target environment) and efficiency (it shows high performance in case of large number of targets). Three examples are considered to demonstrate the feasibility, simplicity, and efficiency of the proposed data association approach in a multisensor-multitarget tracking system.

The remainder of this paper is organized as follows. A brief overview of data association approaches in MSMT tracking systems is given in Section 2. The fuzzy clustering means algorithm is introduced in Section 3. The problem formulation and the proposed fuzzy logic data association approach are presented in Section 4. Performance evaluation and results using embedded Shenyang simulations are reported in Section 5. A comparison with other fuzzy logic data association approaches based on the results described in \(^\text{[16]}\) is also presented in Section 5. A discussion of the computational complexity is presented in Section 6. Section 7 contains conclusions.

## 2 Data association of fusion in tracking systems

In the current literature, there are two main categories of data association in MSMT tracking systems: algorithmic and non-algorithmic\(^\text{[16]}\). The algorithmic category is based on nearest neighbor (NN) and all neighbor (AN) techniques. The non-algorithmic (approximate) category is based on neural network and fuzzy logic techniques. A number of data association approaches exist, and other have been developed to solve the problem of data association in MSMT tracking systems\(^\text{[1-3,8,15]}\).

In the NN approach, one observation, at most, can be used to update a given track. In this approach, measurements are assigned to existing tracks in such a way to minimize/maximize some overall similarity measure. Blackman\(^\text{[8]}\) provides an excellent description of NN association for MSMT tracking. He considered different examples of simple, complex, optimal and sub-optimal distance measures. Bar-Shalom and Fortman\(^\text{[1]}\) developed the probabilistic data association filter (PDAF) and the joint probabilistic data association filter (JPDAF) in which each measurement is assumed to have originated from either a known target or clutter. The result is that the updated estimate for a given track may contain contributions from more than one observation with some association probabilities. The JPDA method is identical to the PDA except that the association probabilities are computed using all measurements and all tracks. A lot of attention has been given to improve the computational efficiency of JPDAF. This can be done by approximating the computation of the association probabilities\(^\text{[10]}\). The performance of an approximation of the JPDAF degrades drastically in case of a dense target.
environment.

Mori et al. describe an example of a multiple hypothesis tracking (MHT) approach. This is recognized as the theoretically best approach for the MSMT tracking problem under idealized modeling assumptions, yet it requires a considerable amount of computation and memory. Outputs from MHT algorithms are typically a list of hypotheses that can be ranked by their probability estimates. Unlike the MHT, which is a hard decision (see relative references) multi-scan data association approach, the interesting multiple model joint probabilistic data association approach, the interesting model joint probabilistic data association is a soft decision (see relative references) zero back scan data association approach, which combines in a probabilistic way several tracks to update the target state estimate. An example of this approach is described in [3]. Molnar et al. described an iterative procedure for time-recursive MSMT tracking based on an expectation-maximization (EM) algorithm. Their algorithm is proved to be effective compared to PDAF and JPDAF at the expense of additional computations.

In general, optimal solutions are not computationally feasible for real-time surveillance system. Furthermore, a priori knowledge of the signal environment is limited in practice. Thus sub-optimal but computationally feasible solutions are developed. A large number of sub-optimal data association approaches in MSMT tracking systems have been developed in the literature.

Unlike the algorithmic category, the non-algorithmic category provides approximate solutions to the problem of data association. Sengupta and Iltis developed an analog neural network to emulate the JPDAF. Their approach is capable of handling six targets and twenty measurements at most. The implementation of their approach is difficult due to the heuristic nature of the approach. Singh and Bailey developed a first fuzzy logic approach for the data association problem. Their approach can be applied to solve data association problems in MSMT tracking. In their approach, the distance measure has not been used in the usual manner, but the fuzzy logic technique has fuzzified the distance measures for use by the fuzzy knowledge-base (IF THEN rules). The major advantage of their approach is its ability to handle different types of information. Unfortunately, the extension of their approach to the case of more than three or four targets is computationally unfeasible due to the large number of rules.

Application of the fuzzy logic approach to the data association problem provides an approximate solution, and the results are subject to the number of input variables, the number of linguistic variables, the membership function, and the accuracy of the rules. Although Sing and Bailey present the critical problem of constructing the optimal membership function for a given distribution of the data, the optimal membership functions are constructed using approximate methods[^9].

3 Fuzzy clustering means algorithm

The most widely used clustering algorithm is the fuzzy clustering means algorithm (FCM) developed by Bezdek[^4,5,7]. This section introduces the FCM algorithm which will be used for measurements-to-tracks association is to classify the data into a number of cluster (groups)[^6,11,13]. The clustering algorithms produce a degree (grade) of membership for each data point in each cluster. Unlike conventional clustering, which involves a partitioning of objects into disjoint clusters, fuzzy clustering allows a data point \( x \) to have a partial degree of membership in more than one set. A fuzzy set \( A \) in a collection of objects \( X \) is defined as \( A = \{(x, u_A(x)) | x \in X \} \) (1) Where \( u_A(x) \) is the degree of membership function of data point \( x \) in fuzzy set \( A \). Given a number of
data points, it is required to group(cluster) the data into clusters according to some similarity measure. Let $c$ be an integer which represents the number of clusters with $2 \leq c \leq 5n$, where $n$ is the number of data points. Define $U$ as a partition matrix of elements $u_{ij}$ ($i=1,2, \ldots, c; j=1,2,\ldots,n$) which represents the degree of membership of data point $j$ in fuzzy cluster $i$, such that 
\[ u_{ik} \in [0,1], \quad 1 \leq i \leq c, 1 \leq k \leq n \]
\[ \sum_{i=1}^{c} u_{ik} = 1 \quad \forall k \quad 0 < \sum_{k=1}^{n} u_{ik} < n \quad \forall i \]  
(2)

Define $J_m$ as the sum of the squared errors weighted by the $m$th power of the corresponding degree of membership, i.e.,
\[ J_m(U, v) = \sum_{k=1}^{c} \sum_{i=1}^{n} (u_{ik})^m (d_{ik})^2 \]  
where \( (d_{ik})^2 = \|v_i - v_k\|^2 \)  
(3)

And $\| \|$ is any inner product induced norm, $m \in [1, \infty]$ is a real number called the fuzzification constant (or weighting exponent), $x_k$ is a data point and $v_i$ is a cluster center. The degrees of membership will be established by minimizing the sum of the squared errors weighted by the corresponding $m$th power of the degree of membership. The goal of the fuzzy clustering algorithm is to determine the optimum degrees of membership $u_{ik}$ $(\forall i,k)$ and the optimum fuzzy cluster centers $v_i$ $(\forall i)$ such that the sum of the square errors $J_m$ is minimum. The results are given by [3].
\[ u_{ik} = \frac{1}{\sum_{j=1}^{c} (d_{jk} / d_{ik})^{2/(m-1)}} \quad \forall i,k \]  
(4)
\[ v_i = \frac{\sum_{k=1}^{n} (u_{ik})^m x_k}{\sum_{k=1}^{n} (u_{ik})^m} \quad \forall i \]  
(5)

Where (4) is valid for a fixed $V(\{v_1, v_2, \ldots, v_c\})$, and solution (5) is valid for a fixed $U$. In MSMT tracking systems, $c$ is the number of targets, $n$ is the total number of received measurements, $x_k$ is the $s$-dimensional measurement vector ($k=1,2,\ldots,n$), and $v_i$ is the $s$-dimensional predicted vector for target $i$. The fuzzy $c$-means clustering algorithm or the Picard algorithm is guaranteed to converge to a local minimum$^{[7,14]}$.

The fuzzification constant $m$ plays an important role. It reduces the influence of noise when computing the degree of membership (4) and the cluster centers (5). The weighting exponent $m$ reduces the influence of a small $u_{ik}$ (for data that are far away from the cluster centers) compared to a larger $u_{ik}$ (for data that are close to the cluster centers). As $m$ increase, its influence becomes stronger. For more details about the weighting exponent, see other probability reference.

4 Proposed fuzzy logic data association approach

Suppose that $n$ measurements are received at time index $t$ (scan $t$). The number of measurements ($n$) does not necessarily equal the number of targets ($c$). It is required to assign (associate) only one measurement can have only one origin. The data association suffers from two kinds of errors: missed correlation and incorrect correlation. In the case of a dense target environment accompanied by noises and other interferences, the data association problem is the most critical problem in tracking systems. Gating techniques cannot solve the problem of association measurements with tracks when a measurement falls within the gates of multiple target tracks or when multiple measurements fall within the gates of a target track. In which case a different technique is required. Our goal is to associate each measurement $x_k$ with one of $c$ possible tracks given predicted values $v_i$. The target’s predicted values $v_i$ can be estimated using optimal filtering techniques, such as least squares, tracker and Kalman filtering techniques. The choice of a particular optimal filtering technique is not arbitrary but depends on the application and the assumed target state model.

The proposed fuzzy logic data association approach consists of the following steps:

1) Apply the FCM algorithm for a fixed $v$
and find the partition matrix $U$, which represents the degrees of membership of all measurements to all tracks. The association matrix $U$ represents the assignment matrix between all observations (measurements) and all entities (targets). Each element in the partition matrix $u_{ik}(i=1,2,...,c; k=1,2,...,n)$ represents an association measure between the predicted value of track $i$ and measurement $k$.

II) Search for the maximum degree of membership $u_{ik\text{-}\text{max}}$ (the closes measurement-to-track pair) and make the indicated assignment, i.e. associate measurement $k$ to track $i$.

III) Remove the measurement-to-track pair identified above from the assignment matrix $U$ and obtain the reduce matrix (this step is a virtual operation that aims to simplify the analysis and does not affect the values of the parameters $u_{ik}$).

IV) Repeat rules II and III for each remaining track until all the $n$ measurements are assigned to the $c$ existing tracks.

V) Obtain the final assignment of measurements to tracks.

To explain the above approach with the help of an example, suppose that there are four targets under surveillance in a given scan ($c=4$) with predicted vectors $v_1$, $v_2$, $v_3$ and $v_4$ and four measurements $x_1$, $x_2$, $x_3$, $x_4$ ($n=4$). We assume that the correct correlation is to assign measurement $i$ to track $i$, $i=1,2,3,4$. Furthermore, assume that we apply the FCM algorithm for the given predicted vectors, using (4), and the partition matrix $U$ is determined as

$$U = \begin{pmatrix}
0.20 & 0.55 & 0.15 \\
0.20 & 0.25 & 0.32 \\
0.70 & 0.15 & 0.80 \\
0.25 & 0.15 & 0.15 \\
\end{pmatrix}$$  (6) where

the rows represent tracks and the columns represent measurements. We have $u_{k\text{-}\text{max}}=u_{33}=0.8$; thus measurement 3 is assigned to track 3. The reduced matrix is given by

$$U = \begin{pmatrix}
0.20 & 0.55 & 0.15 \\
0.20 & 0.25 & 0.32 \\
0.25 & 0.15 & 0.50 \\
\end{pmatrix}$$  (7) In this case $u_{k\text{-}\text{max}}=u_{12}=0.55$; thus measurement 2 is assigned to track 1 and the reduced matrix will be

$$U = \begin{pmatrix}
0.20 & 0.32 \\
0.25 & 0.50 \\
\end{pmatrix}$$  (8) For the reduced matrix we have $u_{k\text{-}\text{max}}=u_{44}=0.50$; thus measurement 4 is assigned to track 4. Finally, the reduced matrix $U_{\text{red}3}$ will have the value $0.2(u_{21})$, and measurement 1 is assigned to track 2. The final assignment of measurements to track is: measurement 1 is assigned to track 2 (incorrect correlation), measurement 2 is assigned to track 1 (incorrect correlation), measurement 3 is assigned to track 3 (correct correlation), and measurement 4 is assigned to track 4 (correct correlation). In this case we performed two correct correlations in a 4-target environment. Thus we performed 50% with respect to perfect correlation in this example.

5 Performance evaluation and comparison

Singh and Bailey[16] proposed the first fuzzy logic approach for the data association problem. They applied their approach to the case of a two-dimensional multisensor-multitarget tracking system. Their approach performed 80% with respect to perfect correlation for the case of two targets with position standard deviations (3.6, 3.5 Ft) and speed standard deviations (0.81, 0.98Ft/s).

Two similar example are considered here to demonstrate the feasibility, simplicity and efficiency of the proposed data association approach in an MSMT tracking system. In our simulation, the distance $d_k$ is calculated using the Euclidean norm, and the degree of membership $u_{ik}$ is calculated assuming $m=2$. It is worth noting that we do not use the FCM
iterations to derive the partition matrix $U$. Instead, we determine $U$ directly from (4) without any iterations. This is because the predicted vectors $v_i$ can be estimated using filtering techniques. Thus we fix $V$ and apply (3) and (4) directly to determine $U$.

5.1 Two-dimensional tracking system

We consider an example of six targets moving with constant acceleration([16] considered the same example for one and two targets), with acceleration $a=0.55Ft/s^2$, and sampling interval $t=1s$. For a given scan $t$, the target trajectories (speeds and positions) are governed by $S_i(t) = S_i(t-1) + aT_i$ (9)

$$\begin{align*}
P_i(t) &= P_i(t-1) + S_i(t)T_i.
\end{align*}$$

The initial positions and speeds of the targets are $[(9,5), (100,4),(30,7),(65,3.5),(45,2),(75,2)]$. The measurement noise is assumed to be Gaussian. The standard deviations of positions are $3.65, 3.70, 3.75, 3.80, 3.90$ and $4.0Ft$ and that of speeds are $0.85, 0.9, 1.0, 1.1, 1.15$ and $1.2Ft/s$. The true positions and speeds are the predicted values. The true and the actual target positions and speeds are fuzzified using the FCM algorithm. The simulation was run for 10s (ten measurement). The final assignments of measurements to tracks are performed using the steps mentioned in Section 4.

Fig.2 depicts the true target trajectories. Fig.3 depicts the true target trajectories along with their sampled positions. All the position values in the figures are given in feet. The correlation results in terms of the degrees of membership are shown in Fig.4 and Fig.5 (dismissed). The $Y$-axis represents the degree of fuzzy correlation variables $R(i, i)=u_{ii}$, $i=1,2,\ldots, 6$. The corresponding binary correlation values (hard data association) are shown in Fig. 6 and Fig.7 (dismissed) show that the proposed approach performs 100% (with respect to perfect correlation) for target 1, 100% for target 2, 100% for target 3, 90% for target 4, 90% for target 5, and 100% for target 6. Thus, in an average sense, the proposed approach performs 96.67% with respect to perfect correlation. It performs 95.3% with respect to perfect correlation over 10,000 Monte simulations.
5.2 Four-dimensional tracking system

We consider the case of moving targets in $x$ and $y$ positions. The example considers the case of four crossing targets with common noise level in $x$ and $y$ positions denoted as noise standard deviation $\sigma$. The targets motion model is assumed to be

$$X(t+1) = FX(t)$$ (10)

$$F = \begin{bmatrix} 1 & \Delta & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ (11)

Where $\Delta$ is the sampling interval and $F$ is the state transition matrix given by (11). The $4 \times 1$ state vector $X(t)$ contains the $x$ and $y$ target positions and velocities, i.e.

$$X(t) = (x(t), v_x(t), y(t), v_y(t))^T$$ (12).

The measurements are the $x$ and $y$ target positions given by

$$z(t) = H(t)X(t) + w(t)$$ (13) Where

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$. The noise sequence $w(t)$ is uncorrelated and Gaussian with zero mean and covariance matrix

$$C_w = Cov(w(t)) = \begin{bmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{bmatrix}$$

where $\sigma^2$ is the variance of the measurement error in both $x$ and $y$ position for all targets. In this example, we assume that $x$ and $y$ positions (measurements) are taken every 0.1s; we process 150 samples. The cluster centers $v_i$ are determined as the predicted target positions using the standard Kalman filter, which is usually implemented in practice. The true target trajectories along with the mean sampled position over 1000 embedded Shenyang simulations[16] are depicted in Fig.8 (dismissed). The results indicate that the proposed approach achieves reasonable performance even in high noise levels.

6 Complexity analysis

The computational complexity measure here is defined in terms of the number of rules used for defuzzification. In conventional fuzzy logic techniques, membership functions are represented by a number of linguistic variables $l$ (such as small, medium, large,). These membership functions map the crisp data into $l$ fuzzy sets which define $l$ linguistic values. We assume that each measurement has dimension $s$, which represents the number of kinematic data and attributes, and the number of targets is equal to the number of measurements ($c = n$) for simplicity. The fuzzification of $s$ input variables, each represented by $l$ linguistic variables, requires $Nr$ IF THEN rules, where [16] $N_r = (lc)^s$ (14)

As shown in Tables 1-3 (dismissed), adding a target (in case of conventional fuzzy logic systems), for a fixed number of rules from 12,500 to 21,322 to 31,132. Adding a linguistic variable, for a fixed number of targets and input variables, increase the number of rules from...
12,500 to 22,848 to 37,025. Adding an input variable, for a fixed number of targets and linguistic variables, increases the number of rules from 12,500 to 400,293 to 8,102,134, which is the worst case. Based on the above observations, we remark that the proposed approach requires fewer computations compared to the existing fuzzy logic data association approaches.

7 Conclusion

For data fusion, the problem of data correlation in MSMT tracking systems has been considered. A fuzzy logic data association approach has been proposed to solve this problem. The proposed new approach is based on the fuzzy clustering means algorithm. The association between measurements and tracks is determined using optimal membership functions derived from the fuzzy clustering means algorithm for fixed predicted vectors. The final assignment of measurements to tracks is completely obtained from the partition matrix between the measurements and the tracks. Embedded Shenyang (of China) simulations have been provided to demonstrate the simplicity, feasibility and efficiency of the proposed approach. Performance evaluation has been done and compared to the perfect correlation and other conventional fuzzy logic data association approaches. The results enable us to conclude that the new approach is more efficient and requires fewer computations than the existing fuzzy logic data association techniques.

References