Bias Effects Using an Interacting Multiple Model Approach for Fixed-Sensor Passive Sonar Surveillance Target Tracking

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Abstract - The Interacting Multiple Model (IMM) algorithm has been shown to balance estimation performance and computational efficiency for certain applications involving time-varying hybrid systems. Such systems require more than one valid process model to completely describe operational behavior over time and switching between process models may occur during the estimation process. One example of a time-varying hybrid system involves the target-tracking problem with potentially maneuvering targets. It has been shown that, given careful design, the IMM algorithm can be successfully applied to this problem in an Air Traffic Control setting, [3]. In this study we consider the utility of this design as applied to the maneuvering target tracking problem in a fixed-sensor passive sonar surveillance setting.

Keywords: Tracking, passive sonar, interactive multiple model, IMM.

1 Introduction

In [3], an Interacting Multiple Model (IMM) algorithm design for tracking maneuvering airborne contacts was considered. The results of this study show excellent tracking performance against both maneuvering and non-maneuvering targets with only modest computational burden. The fixed-sensor passive-sonar surveillance problem is similar to the air traffic control problem. The main difference lies in the accuracy of the sensors with regard to the period in which the target motion is sampled. Additionally, fixed-sensor passive-sonar surveillance systems are often used to cue other anti-submarine warfare (ASW) assets in their search for targets of interest and hence minimizing the area of uncertainty associated with a target position estimate is as important as pure RMS error performance. This paper considers both the application of the IMM algorithm design in [3] to the fixed-sensor passive-sonar surveillance problem and the impact of sensor accuracy on the Interacting Multiple Model (IMM) algorithm.

Historically, fixed-sensor passive-sonar target localization and tracking has been performed using human operators to identify potential target maneuvering conditions and ensure track integrity. Operators have utilized estimation algorithms to extract and refine target information of interest (ie. bearing, frequency, signal to noise ratio) and track targets. The main tool for performing target tracking has been the (extended) Kalman filter. The Kalman filter has been shown to provide an optimal tracking solution when the system model is linear and known [4]. To the extent that target motion is uniform these algorithms are highly successful. When there are multiple models which apply to a system, such is the case with the maneuvering target tracking problem, no single Kalman filter can be used to perform tracking without using unacceptably high levels of process noise. Instead, several filters, one for each applicable model, need to be used.

There are several methods for maintaining multiple Kalman filters in a system. The IMM algorithm method discussed in this paper maintains a single set of filters equal to the number of models in the system whose outputs are combined based on the likelihood the current measurements fit each underlying filter model. This approach has been shown to provide a good tradeoff between estimation performance and computational load [4,5].

Overall tracking performance using the IMM algorithm design in a wide range of maneuvering and non-maneuvering target scenarios was good. However, in numerous simulations using the IMM design, a clear bias in the output target speed and to a lesser extent target position was noted. This paper describes the nature of this bias in this particular design and the impact on system performance.
2 Fixed-Sensor Passive Sonar Surveillance

Fixed-sensor passive sonar systems consist primarily of line arrays deployed upon the ocean floor covering a geographic region. Each sensor in the sonar field detecting the target produces an independent measure of conical angle from the sensor to the target. Each sensor also produces an estimate of conical angle uncertainty based on the underlying beam pattern and the signal to noise ratio (SNR) of the target. The intersection of the conical angle with a nominal target depth range yields a hyperbola indicating the azimuthal estimate of target direction relative to the sensor, which is called bearing. Likewise, the uncertainty in conical angle and target depth is converted into a bearing standard deviation.

Assuming that targets are properly associated across arrays, the intersections of bearings from multiple arrays yield position estimates from which a covariance matrix is calculated using the Fisher Information Matrix technique. The combination of position and covariance matrix estimates is referred to as a posit, which is the input measurement applied to the IMM algorithm.

A simple field consisting of four arrays is shown in Figure 1. Also note that this is the field under consideration for the purposes of simulation and performance analysis in this study. Given long-range, low frequency acoustic propagation, sensors are spaced at significant distances to minimize cost relative to a desired probability of target detection. While cost effective, large sensor separations magnify the effect of seemingly small bearing uncertainties into large positional uncertainties.

Consider the sensor field in Figure 1 with a target starting at the position [-10 km, 10 km] and heading due east, 90° heading relative to true north, at a speed of 5 knots. Given a bearing standard deviation, σ, which varies between 0.3-0.7° depending on the SNR of the target at the receiving sensor, then the positional standard deviations in the x and y directions as a function of time are as shown in Figure 2.

In the example presented in [3], an aircraft with an assumed speed of 120 m/s was tracked using a radar system with average positional uncertainty equal to 100 m at a 5-second sampling period. By contrast, extremely fast moving ocean vessels have an assumed speed of 10-20 m/s with similar positional uncertainty at a 10-second sampling period. This difference will be shown to introduce significant issues with model interactions within the operation of the IMM algorithm.

3 Interacting Multiple Model

This section will provide background on the IMM algorithm and the particular configuration of the underlying state estimation filters, Kalman filters, used in this study. Since complete consideration of the IMM algorithm has been covered in numerous references [1-5], this paper will focus on conceptual operation of the algorithm. The IMM algorithm represents an effective balance between tracking performance and computational efficiency.

3.1 Algorithm Overview

The multiple model approach is a technique for modeling a class of systems called hybrid systems. Hybrid systems are systems that are comprised of multiple, but finite, underlying models, where a single model from the set of system models is in effect at a given time. Hybrid systems can either be time-invariant, in which a single model from the set of system models is in effect during the entire estimation process, or time varying, where the model in effect is allowed to switch during the estimation process. The optimal multiple model approach as applied to the time varying hybrid system problem requires an exponentially increasing number of filters in order to perform the estimation process. This makes the optimal multiple model approach computationally infeasible.
The IMM algorithm is a sub-optimal method for dealing with the exponential increase in the number of state estimation filters required by the optimal multiple model approach. As illustrated in Figure 3, the IMM algorithm maintains multiple state estimation filters, called modes, which operate in parallel. An independent state and covariance matrix estimate for each mode is maintained. Estimates from the IMM algorithm, being the center of this study, is discussed in more detail in section 3.3.

3.2 Selection of Models and Parameters

The system effectively uses four state variables: x and y position, along with x and y components of target velocity. The rate of angular change is held as a constant value for each of the filter modes so it was not included in the filter state estimate.

The design of the IMM used in this study follows the design specified in [3] extended to handle both left and right turns. In this design, two types of Kalman filters are used to model both maneuvering and non-maneuvering target motion. The first filter models uniform target motion using a second-order (constant velocity) kinematic model with zero-mean process noise modeling linear accelerations. This is commonly referred to as the nearly constant velocity motion model. To the extent process noise is used, the associated target area of uncertainty is enlarged. Therefore process noise is kept to the minimum necessary to handle expected linear accelerations. The equation, written in matrix-vector and expanded forms, governing this model is given by:

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \\ \omega_{xk+1} \\ \omega_{yk+1} \end{bmatrix} = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_k \\ y_k \\ \omega_{xk} \\ \omega_{yk} \end{bmatrix} + \begin{bmatrix} T^2/2 \\ 0 \\ \sin(\omega T) \\ \sin(\omega T) \end{bmatrix} \begin{bmatrix} v_x \\ v_y \end{bmatrix}$$
The Markov chain transition probabilities, \( \pi_{ij} \), between the three models were defined to be:

\[
\pi_{ij} = \begin{bmatrix}
0.9 & 0.05 & 0.05 \\
0.075 & 0.9 & 0.025 \\
0.075 & 0.025 & 0.9
\end{bmatrix}
\]

Note that the selection of Markov chain transition probabilities is not exact. Selection of the transition probabilities represents a tradeoff between uniform motion performance and performance during target maneuvers.

In some multiple-model filters, the targets of interest are restricted to a set of models, each of which is implemented as a filter. In this case however, the targets are not assumed to match any of the models. It is assumed however that the target motion is bounded near the +/- 1 degree/second turn rates, so that in all cases at least one of the models can track the target. In many cases the actual target motion will lie between one of the turn models and the nearly constant-velocity model, in which case both models will contribute to the result in a meaningful way.

### 3.3 Model Interactions

Within the IMM system, there are two mixing equations. The first uses the likelihood of each filter mode, \( \mu_i \), to determine a system output state, which would be used by the application. These mixing equations are given as:

\[
x(k|k) = \sum_{i=1}^{N} \mu_i x_i(k|k)
\]

\[
X_i = (x_i(k|k) - x(k|k)) (x_i(k|k) - x(k|k))^T
\]

\[
P_i(k|k) = \sum_{i=1}^{N} \mu_i [P_i(k|k) + X_i]
\]

The second mixing equations are used to produce the input state for each of the state estimation filters at the beginning of each cycle. The input state is determined by a combination of the mode likelihoods and the Markov chain transition probability matrix, \( \pi_{ij} \):

\[
u_{ij} = \pi_{ij} \mu_i(k - 1) / \sum_{i=1}^{N} \pi_{ij} \mu_i(k - 1) \quad (8)
\]

\[
x_{0j}(k|k) = \sum_{i=1}^{N} \mu_{ij} x_i(k|k) \quad (9)
\]

\[
X_i = (x_i(k|k) - x(k|k)) (x_i(k|k) - x(k|k))^T \quad (10)
\]

\[
P_{0j}(k|k) = \sum_{i=1}^{N} \mu_{ij} [P_i(k|k) + X_i] \quad (11)
\]

The result of the input mixing equations is that the state estimate contains information from all modes in the system. This is both true and necessary if the actual target model does not conform to any of the underlying Kalman filter models. It will also be true if the target motion conforms to the model used in one of the Kalman filters, with the magnitude of the effect based on the measurement noise and the modeled process noise for each of the filters. While in this case one filter will be more likely (maybe much more likely) than the others, the others will have some likelihood values which will translate to a small but noticeable effect on the mixed values, both for the output state and the input state of each filter.

In the case of the specific application, we find, as shown in Figure 4, that when a target is moving in a straight line the nearly constant velocity mode becomes more likely than the other modes. The two coordinated-turn models (turning in opposite directions) tend toward an equal but smaller likelihood, one pulling the target toward the left and the other toward the right. The effect of these two filters orthogonal to the direction of motion is to cancel each other out, but the effect in the direction of motion is additive, effectively pulling back on the target state information at a level based on their combined likelihood. This causes a small but perceptible bias in the position value, causing a lagging position.

This effect is more pronounced when the velocity values are inspected. In this case, the velocity vectors projected into the direction of the target are additive to the target, while the velocity vectors in the orthogonal direction cancel each other out, such that the heading of the target is unaffected (for this example). However, since each of the two turn models translates some of the speed in the direction of the target into the orthogonal direction (based on the turn rate), the combination of the three velocity values in the direction of motion is less than the true speed of the target.

Similarly, if the target is turning to the left, the constant-velocity filter and the right-turn filter will both exert some bias which will tend to pull the target position back and to the right, and lower the velocity in the direction of motion. If the target motion does not match any filter, then all filters
will tend to bias the result to lower the velocity and pull back the target position.

4 Simulation Performance

Multiple runs with various maneuvering and non-maneuvering targets were simulated. The IMM design was able to track all targets. Target containment within the 2σ area of uncertainty generated by the filter was consistent with expected statistical containment of the truth. In all simulations the bias as previously explained was observed. To study this bias, special focus was given to a simple straight-line target moving at 5 knots at a heading of 90 degrees within the field. The uniform target motion case was selected to make analysis of the bias and its effect easier.

Figure 4 shows the mode probability of the each of the filter modes for the target run. Mode 1 is the nearly constant velocity filter, while modes 2 and 3 are the left and right turn models respectively. For optimal tracking, the probability of mode 1 should go to one since the underlying motion model of the filter mode matches the actual target motion. Likewise, the probabilities of modes 2 and 3 should go to zero. Instead we observe that the likelihood of mode 1 never exceeds 0.6, while the likelihoods of modes 2 and 3 are significant. Further, it appears that mode 3 has a slightly higher probability than mode 2. This last result is not due to the bias problem being examined, but rather with a measurement noise bias introduced by the topology of the sensors and the relative location of the target. Since the target is significantly closer to a single sensor than the rest of the sensors in the field, the measurement uncertainty tends to be shaped by the bearing uncertainty cone for this sensor. This in essence is coloring the measurement noise and must be dealt with in future studies.

As an example, we can examine the information from a single simulation run. The sums of the innovations for the three filter modes (straight, left, and right) are 5.8228, 9.4393, and 12.8437 respectively. This indicates that, on average, the nearly constant-speed filter estimates were more consistent with the input measurements. However, a comparison of values by step shows that, of the 361 steps in the simulation, the nearly constant velocity filter innovation is the smallest in 168, while filter 2 is the smallest in 104 and filter 3 the smallest in 89. Thus, over half the cases filters 2 or 3 had smaller innovations than the correct filter. The effect was even more pronounced if each filter was fed the truth as its input state at each cycle. A sample run showed that filter 2 had the smallest innovation 174 times, compared to filter 1 with 136 smallest innovations (filter 3 was smallest 51 times).

The resulting target speed estimates are shown in Figure 5. A clear negative bias is apparent in the estimated speed values, as indicated by the mean bias line. This bias is also perceptible in the x-axis position values though to a much lesser extent. The y-axis position is not biased by the mixing effect, but is rather biased by the colored measurement noise as described in the previous paragraph.

The reason for the velocity bias becomes clear when a single sample period is analyzed for each filter. Over a 10-second sample period, the target moves about 26 meters. However, as seen in Figure 2, the measurement uncertainty is in the 50-250 meter range for both the x-axis and y-axis. This means that for any single measurement, the measurement position is on the order of 10 times farther away from the predicted state estimate associated with the correct model as compared to the truth. This is illustrated in Figure 6. To the extent that the measurement uncertainty is large enough to overlap the predicted state estimates of the three filter modes, the filters modes retain significant likelihoods. Also, in this case where the target is obeying a uniform motion model, many measurements are actually closer to the prediction from either the left or right turn model than from the nearly constant velocity model.
It is clear that the small target motion relative to the size of the measurement noise causes this effect. Using the same multiple model system, but with a target speed of 500 knots, the probability of mode 1 approaches unity while the probability of the other two filters stay close to zero, and the velocity bias is non-existent.

As discussed in the model interactions section above, when each of the models are equally likely the models conflict with each other. Figure 7 shows the impact of the two nearly coordinate turn filter modes on system output estimate. The combination significantly contributes to the overall bias observed but does not account for all of the bias. The mismatched modes are also impacting the state estimate of the nearly constant-velocity model directly. Intuitively it would seem that the nearly constant-velocity model should compensate by biasing the target velocity high in order to compensate for the mismatched filters and initially this is what happens. However, as the underlying Kalman filter approaches its steady state gain the nearly-constant velocity is unable to compensate rapidly enough to counteract the effect of the mismatched modes.

5 Conclusions

Although the air traffic control problem and the fixed-sensor passive-sonar problems are similar, the differences in accuracy of the sensor relative to target motion over the sampling interval leads to fundamental differences in the operation of the IMM design. When the measurement uncertainty is large enough to contain the updated state estimates from the modes, the mode likelihood for the mismatched models does not go to zero as desired. Therefore the mismatched models will impact the state estimate of both the true model and the output solution. Further this impact will be model dependent.

In the case examined in this study, the result of this phenomenon is biased state estimates to the extent the motion models conflict. Additionally manipulation of the Markov transition matrix to rapidly adapt to a maneuvering target exacerbates the effect of the mismatched modes on the true mode.

From analysis of the IMM two mechanisms that would help alleviate this problem are improved sensor accuracy and lengthened sampling intervals. For this particular application improved sensor accuracy is not feasible to the degree required to resolve slow moving maneuvering target ambiguities. Increasing the sampling interval is also not feasible since it causes the motion model assumptions to become invalid. Therefore the IMM design as formulated for the air traffic control problem is not easily applicable to the fixed-sensor passive-sonar problem without changes to the operation of the IMM. Initial investigations into utilizing information about the interaction of the internal modes in the IMM show promise and demand continuing research.

6 References


