Synchronized Multi-sensor Tracks Association and Fusion

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Abstract - This paper develops the track association and fusion technique of synchronized multi-sensor tracking system. Via the discretization of linear continuous time system, the correlation among common process noise and measurement noises, but also the measurement noises correlation, are proved. And then in the light of theoretic derivation, the synchronized multi-sensor track association and fusion algorithm is presented. The simulation results express its validity.

Keywords: correlated noise  target tracking  association fusion.

1 Introduction

Because of the limitations of using single sensor for tracking targets (such as accuracy, measurement space, etc.) and some other disadvantages, most of tracking systems are being designed with multiple sensors to improve the system’s tracking performance. Since the concept of data fusion was addressed, the multi-sensor information fusion problem is being noticed by a lot of researchers, and multi-sensor target tracking problem has been investigated in lots of literatures ([2]-[11]), ranging from the distributed and central or hybrid tracking systems, involving the problems about noises independence mutually and also the dependence of corresponding estimation errors. The literatures [4] [6]-[8] all consider the track-to-track association problem when dealing with the multi-sensor data fusion. The [8] takes the effect of common noise on two sensors track fusion into account firstly, but few of them discuss the correlation cases between the system’s process noise and measurement noises mutually. This is unconfirmed to the real world.

This paper, beginning with the discretization of linear continuous time system, has a talk about the reasons of the correlation between process noise and measurement noises mutually. And then the algorithm of track-to-track association and fusion is derived in the case of correlated process noise and measurement noises mutually on the bases of [6]-[7]. Finally, its validity is proved via the Monte Carlo simulation.

2 Formulation of the correlated problems

Firstly, let us consider the following linear continuous time system with Wiener process noise:

$$dx(t) = A(t)x(t)dt + \sigma(t)d\xi(t), \forall t \geq 0 \ (1)$$

where $A(t), \sigma(t)$ are coefficient matrices with appropriate dimensions, $x(t) \in \mathbb{R}^n$ is system state vector, $\xi(t)$ is a zero-mean Wiener process with unitary increment covariance. The system initial state $x(0)$ is a random vector with mean $\overline{x}_0$ with covariance $P_0$.

Assumes there are multiple sensors detecting the above linear continuous time system, the corresponding equations are:

$$dy^{(i)}(t) = C^{(i)}(t)x(t)dt + \chi^{(i)}(t)d\eta^{(i)}(t) \ (2)$$

where $y^{(i)}(t)$ is the $i$th sensor’s measurement vector, $\eta^{(i)}(t)$ is also a Wiener process with zero-mean, unitary increment covariance. $C^{(i)}(t), \chi^{(i)}(t)$ are coefficient matrices with appropriate dimensions. $x(0)$ , $\xi(t)$ , $\eta^{(i)}(t)$ are independent with each other.

Let $\Phi(t,s)$ be the transition matrix corresponding to $A(t)$ , $T$ be the sample time interval. To discrete the above linear continuous time equations( 1 ) and( 2 ), we can obtain:

$$x(k+1) = x((k+1)T) \Delta = F(k)x(k) + v(k) \ (3)$$

$$z^{(i)}(k+1) = y^{(i)}((k+1)T) - y^{(i)}(kT) \Delta = H^{(i)}(k+1)x(k+1) + \nu^{(i)}(k+1) \ (4)$$
In (3) and (4), the time index is omitted for simplicity. where
\[ F(k) = \Phi((k + 1)T,kT) \]
\[ v(k) = \int_{kT}^{(k+1)T} \Phi((k + 1)T,\tau)\xi(\tau)d\xi(\tau) \]
\[ H^{(i)}(k + 1) = h^{(i)}(k + 1)F^{-1}(k) \]
\[ h^{(i)}(k + 1) = \int_{kT}^{(k+1)T} C^{(i)}(\tau)\Phi(\tau,kT)d\tau \]
\[ w^{(i)}(k + 1) = \zeta^{(i)}(k + 1) - H^{(i)}(k + 1)v(k) \]
\[ \gamma^{(i)}(k + 1) = \int_{kT}^{(k+1)T} \chi^{(i)}(\tau)d\chi^{(i)}(\tau) \]
\[ \theta^{(i)}(k + 1) = \int_{kT}^{(k+1)T} C^{(i)}(\tau)\Phi(\tau,s)\sigma(s)d\tau \]

In the above equations (3) (4), \(v(k),w^{(i)}(k + 1)\) can be viewed as the discrete process noise and the discrete measurement noise respectively. According to the features of stochastic integral, one can obtain the following results easily: \(E[v(k)] = 0, E[w^{(i)}(k)] = 0\). The system process and measurement noise’s mean, that is to say, are maintained as before, but the corresponding covariance have been changed as:
\[ Q(k) = \text{cov}[v(k),v(k)] = \int_{kT}^{(k+1)T} \Phi((k + 1)T,\tau)\sigma(\tau)\sigma(\tau)\Phi((k + 1)T,\tau)d\tau \]
\[ R^{(i)}(k) = \text{cov}[w^{(i)}(k),w^{(i)}(k)] = \int_{kT}^{(k+1)T} \Phi((k + 1)T,\tau)\sigma(\tau)\sigma(\tau)\Phi((k + 1)T,\tau)d\tau \]
\[ \delta_{ij} \]
\[ \delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \]
\[ r^{(i)}(k + 1) = \text{cov}[w^{(i)}(k),\omega^{(i)}(k + 1)] = \int_{kT}^{(k+1)T} \Phi((k + 1)T,\tau)\phi(\tau)\phi(\tau)\Phi((k + 1)T,\tau)d\tau \]

That is to say, in a word, after the discretization of liner continuous time system, due to the effect of common process noise, the discrete process noise and measurement noise are correlated mutually.

From the above description, we know that, when one merges multiple sensors’ tracks, considering the independence between process noise and measurement noise just only or the correlation of tracks’ estimation errors is unsufficient. In the following section, we present the algorithm about track-to-track association and fusion through theoretic derivation in the case of correlation between process noise and measurement noises.

3. Track-to-track association and fusion

For simplicity, we consider the following figure (figure 1). The tracking system consists of two sensors, every one has its own information processing device to form local target track estimation, and then these local tracks are communicated to processing center to form a better target track estimate.

![Figure 1. The structure of tracking system](image)

Assuming the sampling time is synchronous strictly, the initial states of the two tracks are \(\hat{x}^1(0|0), P^1(0|0), \hat{x}^2(0|0), P^2(0|0)\) respectively, and they are independent of each other.

The dynamic model of tracking target is:
\[ x(k + 1) = F(x(k)) + v(k) \]
\[ \text{The process noise is a mean zero white noise with covariance } Q(k). \text{ The measurement equations from two sensors are:} \]
\[ z^{(i)}(k) = H^{(i)}(k)x(k) + w^{(i)}(k) \quad i = 1,2 \]
\[ \text{The measurement noise } w^{(i)}(k) \text{ is a white noise with mean zero, covariance } R^{(i)}(k). \text{ The covariance of the two sensors’ measurement noises is:} \]
\[ R^{12}(k) = \text{cov}[w^{(1)}(k),w^{(2)}(k)] \]
\[ \text{The covariance between process noise and measurement noises is respectively:} \]
\[ B^1(k) = \text{cov}[v(k - 1), w^{(1)}(k)] \]
\[ B^2(k) = \text{cov}[v(k - 1), w^{(2)}(k)] \]

From the above equations (6) and (7), we can draw a conclusion: due to the effect of the common process noise, the sensors’ measurement noises are becoming correlated mutually after being processed discretely, although they are uncorrelated in the liner continuous time system. This is also coincident with the real world. The correlated situation between the discrete process noise and measurement noise can be obtained by the following derivation:
\[ B^{(i)}(k + 1) = \text{cov}[v(k), w^{(i)}(k + 1)] = \int_{kT}^{(k+1)T} \Phi((k + 1)T,\tau)\phi(\tau)\phi(\tau)\Phi((k + 1)T,\tau)d\tau \]

**Figure 1. The structure of tracking system**
3.1 Track-to-track association

Let \( \hat{x}_1(k), \hat{x}_2(k) \) be the estimated target tracks obtained from the two local estimator respectively. The state estimate errors based on the local estimator are respectively,

\[
\hat{x}_1^2(k) = x_1^2(k) - \hat{x}_1^2(k) \quad (11)
\]

\[
\hat{x}_2^2(k) = x_2^2(k) - \hat{x}_2^2(k) \quad (12)
\]

where \( x_1^2(k) \) and \( x_2^2(k) \) are the corresponding true target states. Assuming the state estimate errors to be Gaussian, Denote the difference of the two local track estimates as

\[
\hat{\Delta}(k) = \hat{x}_1^2(k) - \hat{x}_2^2(k) \quad (13)
\]

From the above equation, we know obviously that this is the estimate of the difference of the true states

\[
\Delta(k) = x_1^2(k) - x_2^2(k) \quad (14)
\]

If \( \hat{x}_1^2(k) \) and \( \hat{x}_2^2(k) \) are the two track estimates coming from the identical target, the hypothesis test can be denoted by:

\[
H_0 : \Delta(k) = 0 \quad (15)
\]

while the different target, the corresponding hypothesis test is

\[
H_1 : \Delta(k) \neq 0 \quad (16)
\]

In order to test the equation (15) is right, we can carry out the following works. Let

\[
\tilde{\Delta}(k) = \Delta(k) - \hat{\Delta}(k) \quad (17)
\]

Obviously, \( \hat{\Delta}(k) \)'s mean is zero if (15) is right , and its covariance can be obtained by

\[
T(k) = E(\hat{\Delta}(k)\hat{\Delta}(k)^T) = E((\hat{x}_1^2(k) - \hat{x}_2^2(k))(\hat{x}_1^2(k) - \hat{x}_2^2(k))^T) \quad (18)
\]

\[
= P^1(2|k|) + P^2(2|k|) - P^{12}(2|k|) = E(\hat{x}_1^2(k)\hat{x}_2^2(k)^T) \quad (19)
\]

The state estimate based on the local sensor’s measurement information, can be calculated by Kalman filtering.

\[
\hat{x}_i^i(k|k) = F(k-1)\hat{x}_i^i(k-1|k-1) +
K_i(k)\hat{x}_i^i(k-1|k-1) \quad (20)
\]

where \( K_i(k) \) is the corresponding Kalman filtering gain \( i = 1,2 \). Combine (20) with the state estimate errors and rearrange the every terms

\[
\hat{x}_i^i(k|k) - \hat{x}_i^i(k) = [I - K_i^i(k)H_i^i(k)]F(k-1)\hat{x}_i^i(k-1|k-1) + Q(k-1)F(k-1) + K_i^i(k)R_i^2(k|k) \quad (21)
\]

Take into account the effects of process noise and measurement noises etc. Substitutes (21) into equation (19)

\[
P^2(2|k|) = E(\hat{x}_1^2(k)(\hat{x}_2^2(k))) =
[I - K_i^i(k)H_i^i(k)]F(k-1)P^1(2|k-1|)F(k-1) + Q(k-1)F(k-1) + K_i^i(k)R_i^2(k)|k-1|F(k-1)
\]

3.2 Tracks fusion

If the hypothesis \( H_0 \) is accepted, then one can carry out the tracks fusion (combination) of the two estimates.

Consider the static linear estimation that yields the posterior mean \( \hat{x} \) in terms of the prior mean \( \bar{x} \) and the measurement \( z \)

\[
\hat{x}(k) = \bar{x} + P_\infty P^{-1}_\infty (z - \bar{z}) \quad (26)
\]

When carrying out tracks fusion, \( \hat{x}_1^1(k|k) \) can be viewed as the tracking target prior mean, but \( \hat{x}_2^2(k|k) \) be the measurement

\[
P_\infty = E(\bar{x}_1^1(k)(\bar{x}_1^1(k)-\bar{x}_2^2(k))^T) \]

\[
= P^1(1|k) - (I - K_1^1(k)H_1^1(k))F(k-1)P^1(2|k-1|)F(k-1) + Q(k-1)F(k-1)
\]

\[
+ K_1^1(k)R_1^2(k|k) + K_2^2(k)R_2^2(k|k)
\]

\[
- (I - K_1^1(k)H_1^1(k))B_2^2(k)(K_2^2(k))^T
\]

\[
- K_2^2(k)(B_2^2(k))^T(I - K_2^2(k)H_2^2(k))^T \] (27)

\[
P_\infty = E(\bar{x}_1^1(k)-\bar{x}_2^2(k))(\bar{x}_1^1(k)-\bar{x}_2^2(k))^T
\]

\[
= P^1(1|k) + P^2(2|k) - P^{12}(2|k) \quad (28)
\]

If substituting (27) (28) into equation (26), we can get the tracks fusion equations.
\[ \hat{x}(k|k) = \hat{x}^1(k|k) + P_{xz} P_{zz}^{-1} (\hat{x}^2(k|k) - \hat{x}^1(k|k)) \]  
\[ P(k|k) = P^1(k|k) - P_{xz} P_{zz}^{-1} (P_{xz})' \]

where \( P_{xz} \), \( P_{zz} \), \( P_{12} \) can be calculated by the equations (27) (28) and (22) respectively. Compared with the literature [1] algorithm, it is different obviously when calculating the cross covariance term. The reason is that the literature [1] doesn’t take into the effects of process noise and measurement mutually. So, in order to get better state estimate when combining local sensors’ tracks, we should better consider the effects of process noise and measurement noises.

4. Simulation results

Consider the following constant-velocity dynamic system

\[ x(k+1) = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} \frac{T^2}{2} \\ T \end{bmatrix} v(k) \]  
(31)

where \( x(k) = [\text{position}, \text{velocity}]' \) and the sample time interval \( T=5s \). \( v(k) \) is a zero mean white noise with covariance \( Q=0.4^2 \). The initial state \( x_0 \sim N(\bar{x}_0, P_0) \) is random with \( \bar{x}_0 = [1000, 100]' \) and covariance \( P_0 = \text{diag}(100^2, 10^2) \). The initial state is independent of process noise.

Let two sensors detect the position of the moving target. The measurement equation is denoted by

\[ z^i(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(k) + w^i(k) \quad i=1,2 \]

where \( w^i(k) \) is random white noise with mean zero, but their covariance are \( R^1(k) = 80^2 \), \( R^2(k) = 100^2 \) respectively. For simplicity, the correlated cases among the process noise and measurement noise are described by the correlated coefficients. The different correlated coefficients among them are considered by the following.

In the following, via the Monte Carlo simulation runs, the validity of the fusion algorithm is proved in view of the correlated relationship between process noise and measurement noises and compared with proposed algorithm by [1] [6]. All the results is given by the root mean square (RMS) error in position or velocity from \( M \) runs.

\[ e(k) = \sqrt{\frac{1}{M} \sum_{j=1}^{M} (\tilde{x}_{jk} - \bar{x}_{jk})^2} \]

where \( \tilde{x}_{jk} \) denotes the estimate error from run \( j \) at time \( k \), \( M=50 \).

Firstly, consider the mutual coefficients among \( v(k), w^1(k), w^2(k) \) are \( \rho_{vw^1} = 0.03\), \( \rho_{vw^2} = 0.03\), \( \rho_{vw^2} = 0.09 \) respectively in the case of the correlated noises. The results given by figure 2 and figure 3 express that, when the correlated relationship are closer, the algorithm given by this paper is better than the others which between process noise and measurement noise are independent and only the state estimate error is correlated.

![Figure 2. RMS position error over 50 runs](image)

![Figure 3. RMS velocity error over 50 runs](image)

When the mutual coefficients among \( v(k), w^1(k), w^2(k) \) are \( \rho_{vw^1} = 0.05\), \( \rho_{vw^2} = 0.05\), \( \rho_{vw^2} = 0.5 \) respectively. The simulation results are given by figure 4 and figure 5. But when the mutual coefficients among \( v(k), w^1(k), w^2(k) \) are \( \rho_{vw^1} = 0.01\), \( \rho_{vw^2} = 0.01\), \( \rho_{vw^2} = 0.01\) respectively, the results are given by figure 6 and figure 7.

From the figure 2 to figure 7, we can draw a basic concept: when the mutual relationship among the process noise and the measurement noises is closer, the algorithm proposed by this paper is better than the others. But as the lower of this relationship, the simulation results of three algorithms are becoming close with each other.
5 Conclusion

This paper proposes an algorithm of track-to-track association and fusion based on the local track estimates in the case of noises correlated. Via Monte Carlo simulation tests and compared with other algorithm, the validity of this algorithm is proved.

References


