Adaptive Multiple Model Algorithm For Sea
Mines Tracking and Type Identification

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Abstract: A in real time multiple model algorithm accurately tracking freely falling ballistic objects (sea mines, bombs) and identifying their ballistic parameters is proposed in the paper. This aim is achieved by by deriving a new discrete model of the objects ballistic and applying the developed here adaptive multiple model estimator. The obtained results illustrate the algorithm efficiency and accuracy.1

Key words: multiple model estimation, extended Kalman filter, ballistics, sea mines

1 Introduction
Relatively cheap and hard for detection and neutralization the sea mines (SM) are the most effective anti-ship and anti-submarine weapon. The standard method of their implementation assumes formation of compact SM areas defending or blocking seaports, breaking sea communications in open sea or in immediate vicinity to the coastline, or preventing possible debarkation on the land. The SM may be delivered to their destination by ships or submarines, but the war experience accumulated beginning from the 60’s shows that the air transport is the fastest and securest way to put SM in vicinity to the enemy forces. They are dropped by aircrafts flying on low height (1-2) km, at speed of (100-250) m/s.

The respective countermeasures are based on continuous surveillance of important air and sea areas by surveillance radars. The SM location is estimated by tracking each mine on the stage of its freely ballistic motion. The possible SM underwater location is assessed by simple extrapolation of its underwater motion. The accuracy of this extrapolation depends on the estimation accuracy of the SM motion parameters, at the splash point (the point of first mine contact with the sea surface). Despite of usually high sensor sampling rate, just few measurements are usually made. Another few ones are made observing the water splash. Evidently, the short time of life of such a ballistic trajectory of (10-20) s, as well as its non-linear nature and unknown ballistic parameters make the considered problem
not easy to be solved by using 2-D surveillance radars. The described in the available references [1, 2] solutions of this problem are focused mainly on an off-line batch processing (interpolation). One-dimensional cubic splines approximate the SM motion along Cartesian axes. Their nodes do not exactly coincide with the radar measurements. These smoothing splines are next considered as solution of the system of the differential equations describing the SM ballistics, identifying (by using the least-square method) the ballistic parameters.

The proposed here approach assumes application of 3-D surveillance radar and real-time estimation of the observed ballistic trajectory by using the MM approach. Because of the relative permanency of the ballistic parameters they are usually known for different kinds of SM, so a full set of mutually contradicting hypotheses about the SM true parameters, may be built. The corresponding set of extended Kalman filters (EKF) based on a common model of the SM fly at different ballistic parameters should “cover” the variety of different kinds of SM. At each scan the output estimate is generated as weighted sum of the separate EKF-s estimates, multiplied by computed in parallel corresponding probability of each SM model (hypothesis) being correct.

The paper consists of five sections. Section 2 describes the ballistics of freely falling object; Section 3 describes the EKF and the respective MM estimator (MME), and Section 4 present results from algorithm performance evaluation by Monte Carlo simulation. Inferences about the algorithm efficiency are given in Conclusions.

2 Derivation of a SM Ballistic Model

The ballistic motion of freely falling object is described by the equations [7, 8]:

\[
\ddot{R} = - \frac{F_x}{m} \frac{\dot{R}}{W} - g + R \ddot{\Lambda}^2 ;
\]

\[
\dot{\Lambda} = - \frac{F_x}{m} \frac{\dot{\Lambda}}{W} - 2 \frac{\dot{R} \Lambda}{R} ,
\]

where \( R \) is the radius-vector of the object location in geocentric coordinate system (Fig. 1), \( \Lambda \) is the angle enclosed between this vector and axis \( OY \), \( W \) is the object air speed, \( m \) is its mass. Except on the changing air speed, the drag force \( F_x = C_{x0} \frac{p W^2}{2} S \) depends on few constant parameters: the drag coefficient \( C_{x0} \), the air density \( p = 1.2257 \text{[kg/m}^3\text{]} \) - practically constant for the considered range of heights, the object’s surface \( S \), \( g = 9.814 \text{m/s}^2 \) - the gravitational acceleration. The object’s horizontal and vertical coordinates \( X \) and \( Y \) are defined as \( X = R \sin \Lambda \) and \( Y = R \cos \Lambda \).

![Fig. 1](image-url)
The above system is usually presented by a differential system of first order:

\[ \dot{R} = w, \]
\[ \dot{w} = -w C_{x_0} \frac{\rho \sqrt{w^2 + (R \omega)^2}}{2m} S - g + R \omega^2, \]
\[ \dot{\omega} = \omega, \]
\[ \dot{\omega} = -\omega \left( C_{x_0} \frac{\rho \sqrt{w^2 + (R \omega)^2}}{2m} S - 2 \frac{w}{R} \right), \]

where \( w \) and \( v \) are vertical and horizontal components of the aircraft’s air speed \( W = \sqrt{w^2 + v^2} \).

Next, the state vector of the above system is brought to a more standard form \( x = (X \ v \ Y \ w \ Z) \), written in the Cartesian coordinate system \( XOYZ \) (axis \( OZ \) is perpendicular to the \( XOY \) plane and points to the reader). State components \( \omega \) and \( \lambda \) are substituted by new ones \( v, Y \) and \( Z \). This is done by (a) setting \( v = R \omega \) (i.e. \( \omega = \frac{v}{R}, R = \frac{v}{\omega} \)), (b) multiplying both sides of the forth equation by \( R = Y + R_g \), and (c) taking into account that:

\[ \dot{v}_{\text{hor}} = R \omega + \dot{R} \omega = R \omega + w \omega : \]
\[ \dot{X} = v \sin \psi_0, \]
\[ \dot{Y} = w, \]
\[ \dot{Y} = w, \]
\[ \dot{w} = -w C_{x_0} \frac{\rho \sqrt{w^2 + v^2}}{2m} S + \frac{w v}{Y + R_g}, \]
\[ \dot{w} = -w C_{x_0} \frac{\rho \sqrt{w^2 + v^2}}{2m} S - g + \frac{v^2}{Y + R_g}, \]
\[ \dot{Z} = v \cos \psi_0, \]

where \( R_g = 6378245 m \) - the Earth range;
\( W_0, \psi_0 \) - the estimated aircraft velocity and heading in \( t = 0 \), when the SM is begins its ballistic flight. The initial state vector is known:

\[ x_0 = \left( X_0 \ W_0 \ Y_0 \ 0 \ Z_0 \right). \]

This system is used in further in section 5 for true SM trajectory simulation.

Neglecting the small last terms in the second and forth equations, and denoting

\[ p = C_{x_0} \frac{\rho}{2m} S \] it is obtained:

\[ \dot{X} = v \sin \psi_0, \]
\[ \dot{Y} = w, \]
\[ \dot{W} = -w \left( p \sqrt{w^2 + v^2} - g \right), \]
\[ \dot{Z} = v \cos \psi_0. \]

The numerical simulations in Appendix A show 100% coincidence between solutions obtained solving systems (1) and (2) by Runge-Kutta algorithm of fourth order. That is why the EKF model is derived here from system (2).

The next step is to discretize system (2). The exact solution of the second and forth equations of system (1) may be used taking into account that:

\[ \int \frac{dx}{x \sqrt{x^2 + 1}} = \ln \left| 1 - \sqrt{\frac{x^2 + 1}{x}} \right|. \]

Unfortunately, the obtained solutions are non-linear and do not express recursively the above state components as a function of the previous state vector components. That is why the applied here approach uses the
results shown in Appendix A. It assumes that $W_{k-1} = \sqrt{w_{k-1}^2 + v_{k-1}^2} = \text{const}$, so the derived discrete system takes the form:

$$X_k = X_{k-1} + T v_{k-1} \sin \psi_0,$n
$$v_k = v_{k-1} e^{-(p_1 + \Delta p_1) T w_{k-1}},$$

(3) \quad Y_k = Y_{k-1} + T w_{k-1}, \quad w_k = w_{k-1} e^{-(p_1 + \Delta p_1) T w_{k-1}} - T g,

$$Z_k = Z_{k-1} + T v_{k-1} \cos \psi_0,$$

$$\Delta p_k = \Delta p_{k-1},$$

where $T$ is the radar sampling interval and $x_0 = (X_0 \ W_0 \ Y_0 \ 0 \ Z_0 \ 0)$. The last equation augments system (3) providing on-line adjustment of the unknown ballistic parameter $p$ and generally better trajectory estimation (this idea is described in [4]-[6]).

The deterministic simulation (Fig. B1 – B4) shows a perfect coincidence between solutions obtained from models (1) and (3), so model (3) is next used for EKF derivation and of an MME capable to track the SM ballistic motion, identifying its ballistic parameters.

3 A MM Estimator for SM Tracking and SM Type Identification

The MME uses a set of $N$ EKF based on a common SM trajectory model (3), written here in the general form:

$$x_{k/k-1} = f(x_{k-1/k-1}, p) + \xi_{k-1}.$$

The system noise $\xi_k \sim N(0, Q_k)$ is white, and the scalar ballistic parameter $p$ is different in different filters.

At moment $t_k$ each EKF estimates the state vector $x_k = (X_k \ v_k \ Y_k \ w_k \ Z_k \ \Delta \phi_k)$ upon the base of 3-D noisy polar measurements “range-bearing-elevation” $y_k = [r_k, \beta_k, \varepsilon_k]$. Each EKF uses the standard equations:

$$\hat{x}_{k/k-1} = f(\hat{x}_{k-1/k-1}, p),$$

$$P_{k/k-1} = f_{k/k-1} (\hat{P}_{k-1/k-1} (f_{k-1}')) + Q_{k-1},$$

$$S_k = HP_{k/k-1}H' + R,$$

$$K_k = P_{k/k-1}H'S_k^{-1},$$

$$\gamma_k = y_k - H\hat{x}_{k/k-1},$$

$$\hat{x}_{k/k} = \hat{x}_{k/k-1} + K_k \gamma_k,$$

$$P_{k/k} = P_{k/k-1} - K_k S_k K_k',$$

$$\lambda_k = N[\gamma_k; 0, S_k],$$

where $\hat{x}_{k/k}$ and $\hat{x}_{k/k-1}$ are filtered and predicted state vector estimates; $\gamma_k$ and $S_k$ are the filter residual process and its covariance; $P_{k/k}$ is the state vector estimation errors covariance matrix, $K_k$ is EKF filter gain vector, $\lambda_k$ is the EKF likelihood; $f_{k/k-1}^\bot$ is the Jacobi matrix:

$$f_{k/k-1} = \frac{\partial f(\cdot)}{\partial \hat{x}_{k-1/k-1}} = \begin{bmatrix} 1 & T \sin \psi_0 & 0 & 0 & 0 & 0 \\ 0 & f_v & 0 & f_w & 0 & f_p \\ 0 & 0 & 0 & 1 & T & 0 \\ 0 & f_v & 0 & f_w & 0 & f_p \\ 0 & T \cos \psi_0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

where $f_v = -p T \frac{v_{k-1}}{W_{k-1}} e^{-(p_1 + \Delta p_1) T w_{k-1}},$

$$f_w = -p T \frac{w_{k-1}}{W_{k-1}} e^{-(p_1 + \Delta p_1) T w_{k-1}},$$

and where
\[ f_p = -TW_{k-1}e^{-(\rho + \Delta p)\mu_{k-1}}. \]

All EKF use the initial conditions taken from the aircraft state vector and its estimation errors covariance matrix:
\[ x_0 = \begin{pmatrix} \hat{x}_0 & \hat{W}_0 & \hat{Y}_0 & \hat{Z}_0 \end{pmatrix}^T; \]
\[ P_{0,0} = \text{diag}\{\sigma_x^2, \sigma_y^2, \sigma_z^2, \sigma_\beta^2\}. \]

Mapping the polar measurements \[ y_k = [r_k, \beta_k, \epsilon_k] \] in Cartesian ones \[ y_k = [X_k, Y_k, Z_k]; \]
\[ X_k = r_k \cos \epsilon_k \sin \beta_k, \]
\[ Y_k = r_k \sin \epsilon_k, \]
\[ Z_k = r_k \cos \epsilon_k \cos \beta_k, \]
the sensor model becomes linear:
\[ y_k = Hx_k + \xi_k = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} x_k + \xi_k. \]

The covariance matrix of the white, measurement noise \[ \xi_k \sim N(0, R_k) \] is computed at each scan, as follows [9]:
\[ R_k = \begin{bmatrix} r_{xx} & r_{xy} & r_{xz} \\ r_{yx} & r_{yy} & r_{yz} \\ r_{zx} & r_{zy} & r_{zz} \end{bmatrix}, \]
where:
\[ r_{xx} = \sigma_x^2 \cos^2 \epsilon_k \sin^2 \beta_k + \sigma_\beta^2 r_k^2 \cos^2 \epsilon_k \cos^2 \beta_k + \sigma_\epsilon^2 r_k^2 \sin^2 \epsilon_k \cos^2 \beta_k, \]
\[ r_{yy} = \sigma_\epsilon^2 \sin^2 \epsilon_k + \sigma_\beta^2 r_k^2 \cos^2 \epsilon_k, \]
\[ r_{zz} = \sigma_\epsilon^2 \cos^2 \epsilon_k \cos^2 \beta_k + \sigma_\beta^2 r_k^2 \cos^2 \epsilon_k \sin^2 \beta_k + \sigma_\epsilon^2 r_k^2 \sin^2 \epsilon_k \cos^2 \beta_k, \]
\[ r_{xz} = r_{zx} = (0.5 \sin 2 \beta_k) \times \left( \sigma_\epsilon^2 \cos^2 \epsilon_k - \sigma_\beta^2 r_k^2 \cos^2 \epsilon_k \sin^2 \epsilon_k \right), \]
\[ r_{xy} = r_{yx} = (0.5 \sin 2 \epsilon_k) \left( \sigma_\epsilon^2 - \sigma_\beta^2 r_k^2 \sin^2 \beta_k \right), \]
\[ r_{yz} = r_{zy} = (0.5 \sin 2 \epsilon_k) \left( \sigma_\epsilon^2 \cos^2 \epsilon_k - \sigma_\beta^2 r_k^2 \cos^2 \epsilon_k \sin^2 \beta_k \right), \]
\[ \sigma_\epsilon, \sigma_\beta \text{ and } \sigma_\beta \text{ - measurement deviations.} \]

The output MM estimate \[ \hat{x}_{k|k}^0 \] is generated at each scan as weighted sum of the separate EKF's estimates \[ \hat{x}_{k|k}^j \], multiplied by computed in parallel corresponding model probability \[ \mu_k^j \]:
\[ \hat{x}_{k|k}^0 = \sum_{j=1}^{N} \hat{x}_{k|k}^j \mu_k^j, \]
where:
\[ \mu_k^j = \Pr\{p_j | y_k\} = \frac{\mu_k^j \lambda_j^k}{\sum_{i=1}^{N} \mu_k^i \lambda_j^i}, \]
\[ \mu_0^j = N^{-1}. \]

### 4 Algorithm Performance Evaluation

The proposed MME is evaluated by Monte Carlo simulation. Data from 30 independent runs per scenario are accumulated and the state vector mean errors (ME) and the root-mean squared errors (RMSE) are computed.

The known SM types’ parameters are given in Table 1.

Two scenarios have been used. The first one uses known SM type SM3; the simulated true parameter in the second one is \[ p = 0.5 (SM_3 + SM_4) \].

<table>
<thead>
<tr>
<th>SM</th>
<th>( m, kg )</th>
<th>( S, m^2 )</th>
<th>( C_{x_0} )</th>
<th>( p, m^{-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>SM_1</td>
<td>100</td>
<td>0.20</td>
<td>0.26</td>
<td>0.00031</td>
</tr>
<tr>
<td>SM_2</td>
<td>250</td>
<td>0.28</td>
<td>0.36</td>
<td>0.00068</td>
</tr>
<tr>
<td>SM_3</td>
<td>500</td>
<td>1.32</td>
<td>0.64</td>
<td>0.00104</td>
</tr>
<tr>
<td>SM_4</td>
<td>1000</td>
<td>3.14</td>
<td>0.80</td>
<td>0.00154</td>
</tr>
</tbody>
</table>

In both scenarios: \[ X_0 = 25 \text{ km}, \]
\[ Y_0 = 2 \text{ km}, \quad Z_0 = 0 \text{ km}; \quad p_0 = 0, \]
\( W_0 = 250 \frac{m}{s}, \quad \psi_0 = 45^\circ, \quad \sigma_r = 50[m], \)

\( \sigma_{\beta} = \sigma_{\epsilon} = 1^\circ; \quad T = 1s. \)

The appropriate matrix \( Q \) is:

\[
Q = \text{diag}\{0 \quad 100 \quad 0 \quad 100 \quad 9e \cdot 08\}.
\]

The results show a perfect SM type identification and estimation accuracy for known SM (see Appendix C). The results slightly worse in opposite case, when the MME need time for SM type identification.

5 Conclusions

An adaptive multiple model algorithm for freely falling ballistic objects (sea mines, bombs) tracking is developed in the paper. This is done by deriving a new discrete model of the objects ballistics and by designing an augmented multiple model estimator. Four models, utilizing available information about \( C_{x0} \) as models discern parameter, are developed. Because of some uncertainty about this parameter, additional discrimination value \( \Delta P \) is introduced. The estimation of \( \Delta P \) is very coarse and it can not be used for precise identification of the unknown parameter \( C_{x0} \). Newer the less, the use of additional discrimination value \( \Delta P \), made possible to shift the topical model toward the through one, providing high quality of the other estimated components of the state vector. In the same time, the dominance of one of the posterior probability give on excellent opportunity to make a conclusion about the possible mine parameter. The obtained results illustrate a high algorithm efficiency and accuracy sufficient for neutralization of these objects by air-defense system.

References


Appendix A: Examples of the SM ballistics

Fig. A-1

Fig. A-2

Fig. A-3

Fig. A-4

Appendix B: SM ballistic models comparison

Fig. B-1

Fig. B-2

Fig. B-3

Fig. B-4
Legend to Appendix A and B:

‘1’ \[ Y_0 = 2000 \ m, \ W_0 = 250 \frac{m}{s}, \ m = 500 \ kg \];

‘2’ \[ Y_0 = 1000 \ m, \ W_0 = 250 \frac{m}{s}, \ m = 500 \ kg \];

‘3’ \[ Y_0 = 2000 \ m, \ W_0 = 150 \frac{m}{s}, \ m = 500 \ kg \];

‘4’ \[ Y_0 = 2000 \ m, \ W_0 = 250 \frac{m}{s}, \ m = 250 \ kg \];
magenta – results obtained by the discrete model.

Appendix C: Performance evaluation

Fig. C-1

Fig. C-2

Fig. C-3

Fig. C-4

Fig. C-5

Fig. C-6

Fig. C-7