The Fuzzy principle states that everything is a matter of degree. So far many business production problems are solved by Operational Research Optimization Techniques, under the considerations of some assumptions. In the current literature, still we have several applications of fuzzy linear, integer, goal and other programming applications. The main aim of this study is to add new application to the literature and to solve the refinery production problem by using the fuzzy principles. In application, the real refinery model has been developed and an alternative fuzzy model solutions criticized to determine which one is better than the others. Finally, comparing the classical solution by the one of the obtained best solution of the fuzzy models, one can obtain more suitable output of the models than traditions.

**Keywords:** Fuzzy consideration, Multivalued Logic, Fuzzy Linear Programming, Fuzzy Optimization.

**1-INTRODUCTION**

Since Zadeh [1] introduced the concept of possibility, it is realized that a type of impression can be expressed by a possibility distribution. The possibility space which is called pattern space, has been discussed in detail by Nahmias [2] and Sugeno [3]. On the other hand, fuzzy arithmetic with fuzzy numbers or operations of fuzzy numbers by the extensions principle [4,5,6] have been studied in many studies.

Since the pioneer work on fuzzy linear programming (FLP) by Tanaka and Zimmerman, in the last past years several kinds of FLP applications have appeared in the literature and obviously with them different approaches of resolution have been proposed too [7]. All mathematical programming techniques, such as LP, IP, DP and the others have the same objective: maximization or minimization, generally optimization. But in a reality, since everything is matter of degree, the coefficient of objective functions or the values of other components such as right hand side values of model may be considered as a multivalence. It means three or more options, perhaps an infinite spectrum of options, instead of just two extremes.

The structure of the conventional linear programming (LP) problem

\[
\text{Max } cx \\
\text{Subject to } Ax \leq b, \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)
\]

where \( A \) is an \((m,n)\) matrix, \( m \leq n \), and \( c \), \( x \) \( \in \mathbb{R}^m \), \( b \) \( \in \mathbb{R}^m \) columns vectors. As we do know from the literature that the first approach to the Fuzzy Linear Programming (FLP) made by Bellman and Zadeh[8]. After this study, many different types of FLP applications apparied on this area, like critical discussions on membership functions for fuzzy linear programming problems.[9].

The fuzziness of the Classical Linear Programming problems can appear either in the constraint set or in the coefficients taking part in the definition of this set. Namely, we may consider the Fuzzy Linear
Programming Problems, mainly, in two alternatives: FLP problems with fuzzy constraints and FLP problems with fuzzy coefficients.

1.1 Fuzzy LP problems with fuzzy constraints

The base of the FLP problems with fuzzy constraints depends on the tolerance of violation in the accomplishment of the constraints. In other words the decision makers permits the constraints to be satisfied “as well as possible” for each constraint in the constraint set.

This assumption can be represented by

\[ a_i \chi < b_i, \quad i = 1,2,3,\ldots,m \]

and modeled by means of a membership function

\[ \mu_i(x) = \begin{cases} 1 & \text{if } a_i x \leq b_i, \\ f_i(a_i x) & \text{if } b_i \leq a_i x \leq b_i + \delta_i, \\ 0 & \text{if } a_i x \geq b_i + \delta_i \end{cases} \quad (2) \]

Those kind of membership functions expresses that the decision maker tolerates violations in each constraints up the value \( b_i + \delta_i \), \( i=1,2,\ldots,m \). At the same time \( f_i \) functions are assuming as a non-decreasing and continuous. The graphical representation of the situation as shown in the following Figure 1.

![Figure 1: Graphical Representation of \( \mu_i(x) \)](image)

After this definition, the associated problem is usually represented by the following form of the LP structure.

\[ \begin{array}{cc}
\text{Max } c x \\
\text{St } & A x \leq b, \\
& x \geq 0
\end{array} \quad \text{(3)} \]

So far, many different approaches has been developed to solve the fuzzified model on different LP problems [10,11,12]. Fuzzy solution to (3) can be obtained involving as particular values and is found from the solution of the parametric LP problem.

\[ \begin{array}{cc}
\text{Max } c x \\
\text{s.t } & A x \leq g(\alpha), \\
& x \geq 0, \alpha \in (0,1]
\end{array} \quad \text{(4)} \]

where \( g(\alpha) \) is a column vector defined by the inverse functions of the \( f_i, i=1,2,\ldots,m \). Here, the linearity characteristic of (3) is preserved and, in particular, if the \( f_i \) are linear, the new form of the above system (4) becomes such as below:

\[ \begin{array}{cc}
\text{Max } c x \\
\text{s.t } & A x \leq \delta_i + \delta_i (1 - \alpha), \\
& x \geq 0, \alpha \in [0,1]
\end{array} \quad \text{(5)} \]

1.2 FLP Problems with fuzzy coefficients

In many applications, the decision makers do not know exactly the values of the coefficients taking part in a certain problem and, moreover, that vaguness is not of a probabilistic kind, one can model those inexact values by means of fuzzy numbers.

In this situation, the FLP system may be represented as following

\[ \begin{array}{cc}
\text{Max } c x \\
\text{St } & \bar{a}_j x \leq \bar{b}_i, \quad i = 1,2,3,\ldots,m, \\
& x_j \geq 0, \quad j = 1,2,3,\ldots,n,
\end{array} \quad \text{(6)} \]

where \( \bar{a}_j, \bar{b}_i \in N(\mathbb{R}) \), \( j = 1,2,\ldots,n \), are defined by respective membership functions \( \mu_q \) and \( \mu_i \).

2. Definition and Solution of The Production Problem

The application area of this research belongs to a certain production problem on petro-chemistry industry. The production problem which is modeled in a LP structure has 44 variables. Refinery model has 4 different crude oil inputs: Kerkük, Light Iran, Light Arab and Domestic production. The developed Refinary LP model has also 36 constraints which denotes capabilities of market and current system. The full formulated case of the
production problem and its restrictions such as below:

The objective function depending on the variables which are described above is as below:


Here are 36 constraints of the model described unit by unit such as follows:

(1) \( X_1 \leq 800.000 \text{ Ton/Month}, \) Available Kerkuk
(2) \( X_2 \leq 500.000 \text{ Ton/Month}, \) Available L.Iran
(3) \( X_3 \leq 300.000 \text{ Ton/Month}, \) Available L.Arab
(4) \( X_4 \leq 100.000 \text{ Ton/Month}, \) Available Domestic

Unit 100

(5) \( X_1 + X_2 + X_3 + X_4 \leq 909.000 \text{ Ton/Month}, \) Capacity of Unit 100
(6) \( 0.012X_1 + 0.013X_2 + 0.09X_3 + 0.073X_4 = 19530 \text{ ton}, \) From Crude O.to L.P.G.
(7) \( 0.196X_1 + 0.185X_2 + 0.166X_3 + 0.020X_4 = 301100 \text{ ton}, \) From Crude O.to Naphtha
(8) \( 0.115X_1 + 0.118X_2 + 0.115X_3 + 0.109X_4 = 196400 \text{ ton}, \) From Crude O.to Kerosine
(9) \( 0.241X_1 + 0.219X_2 + 0.241X_3 + 0.196X_4 = 394200 \text{ ton}, \) From Crude O.to Motorine
(10) \( 0.436X_1 + 0.465X_2 + 0.469X_3 + 0.487X_4 = 770700, \) From Crude O.to Atm.Rezidu

Unit 200

(11) \( X_{10} + X_{11} + X_{12} + X_{13} + X_{14} = 117429 \text{ ton naphtha}, \) as a input to unit 200
(12) \( 0.41X_5 = 123451 \text{ ton-Naphtha- Final Product} \)
(13) \( X_{15} + X_{16} = 118113 \text{ ton input (H2 and Naphtha) to unit 300} \)

Unit 400

(14) \( X_{17} + X_{18} = 64641.58 \text{ ton input (H2 and Kerosine); to unit 400} \)

Unit 600

(15) \( 0.49X_{11} + X_{34} = 31572.521 \text{ ton LPG input to unit 600} \)
(16) \( X_{19} \leq 6314.5 \text{ ton Propane final product} \)

(17) \( X_{20} + 0.51X_{11} \leq 31845.782 \text{ ton LPG final product} \)
(18) \( X_{13} + X_{15} + X_{17} + X_{33} \leq 6384.4273 \text{ ton Sulphur Unit 1200} \)
(19) \( X_{25} + X_{26} \leq 52226.696 \text{ ton Vacum Rezidu} \)

Unit 1300

(20) \( X_{27} + X_{28} \leq 77746.021 \text{ ton Subproduct of Dist. And Brt.Dist} \)

Unit 1400

(21) \( X_{29} + X_{30} \leq 42908.028 \text{ ton Rafinates input} \)

Unit 1500

(22) \( 0.10X_{10} + X_{29} \leq 33586.858 \text{ ton Final products of M.Oils} \)

Unit 1100

(23) \( X_{22} + X_{23} + X_{24} \leq 303373 \text{ ton Atm.Rez. input} \)
(24) \( 0.035X_8 + 0.61X_9 + 0.20X_{23} + 0.30X_{24} + 0.54X_{28} + 0.36X_{37} \leq 564494.05 \text{ ton Fuel oil} \)
(25) \( 0.697X_7 + 0.007X_8 + 0.30X_{24} + X_{25} + 0.05X_{28} \leq 73481.542 \text{ ton Final products of Asphalt} \)

Unit F.F.C.

(26) \( X_{22} + 0.22X_{23} + 0.30X_{25} + 0.41X_{28} + X_{30} \leq 120205.23 \text{ ton Subproducts, H2 S.L.P.G.,} \)

(27) \( 0.098X_{11} + 0.20X_{34} - X_{19} = 0 \text{ Final product Prophane} \)
(28) \( 0.51X_{11} + X_{30} - X_{12} = 0 \text{ Final product LPG} \)
(29) \( 0.41X_6 - X_{19} = 0 \text{ Final Product Naphtha} \)
(30) \( X_{14} + X_{18} - X_{32} = 0 \text{ Final Product of Kerosine} \)
(31) \( X_{12} + X_{14} + X_{35} - X_{43} - X_{44} = 0 \text{ Final Product of Normal and Super Gasoline} \)
(32) \( 0.65X_1 + 0.943X_6 - X_{41} = 0 \text{ Final Product Motorin} \)
(33) \( 0.05X_6 + 0.61X_9 + 0.20X_{23} + 0.30X_{24} + 0.54X_{28} + 0.36X_{37} - X_{30} = 0 \text{ Final product F.Oil} \)
(34) \( X_{15} + X_{17} + X_{18} - X_{31} = 0 \text{ Final Product Sulphur} \)
(35) \( 0.10X_{10} + X_{29} - X_{31} = 0 \text{ Final Product M.Oils} \)
(36) \( 0.007_4 + 0.30X_{24} + X_{26} - X_{32} = 0 \text{ Final Product Asphalt} \)

The method which is using for formulation of this problem is that the total amounts of input and output at each unit are equal. Some of the coefficients of objective function are positive and negative. The coefficients of first four variables which represent the crude oils and the subproducts, are negative because of cost values. Positive coefficients belonging to the final products are sale prices for per ton. It has been also used the productivity information on formulation of Unit 100. As a decision variable \( X_1 \) (Kerkük Crude Oil) and \( X_3 \) (Light Arab) have been chosen by normal solution. On the other hand, optimum value of the
objective function, determined by selected variables, is as below;

\[ K = \$ 950125.89 \text{ /Month} \]

The sensitivity analysis results of the objective function to coefficients for Kerkük Crude Oil and Light Arab are between (-2945 - 344) and (-348 - Infinite) respectively. According to the normal solution, maximum value of prices of two kind of crude oils $54.1247/Ton and $52.5702/Ton respectively. The other mean of the sensitivity analysis indicates that the crude oils must be bought as cheap as possible.

3- Fuzzy Solutions

Instead of conventional solutions of LP problems, since the real life applications have dynamic structure, the decision makers have to consider alternative models with fuzzy constraints or fuzzy coefficients. To transform the original model to the alternative fuzzy models, the first five constraints, displayed above, has been involved in fuzziness operation.

Since the first five constraints have been considered \( t_i \) values as a fuzzy numbers fixed by decision maker, determines maximum violation in the accomplishment of the \( i^{th} \) constraint. Thus it makes sense to change that \( i^{th} \) constraint by the following one:

\[ a_i x < b_i + t_i (1- \alpha) \quad i = 1,..5, \alpha \in [0,1] \]

which expresses that for \( \alpha = 1 \) the constraint is completely verified with respect to the wishes of the decision maker. Moreover, the smaller \( \alpha \) is, the smaller the accomplishment degree for the decision maker shall be [7].

By following this logic, the triangular fuzzy values of coefficients of the model, introduced above, for first five coefficients, decided such as follows:

\[ \tilde{I}_1 = (1.0,5.1.5) \]
\[ \tilde{I}_2 = (0.000,700.000,900.000) \]
\[ \tilde{I}_3 = (1.0,5.1.5) \]
\[ \tilde{I}_4 = (0.000,400.000,600.000) \]
\[ \tilde{I}_5 = (1.0,75,1.25) \]
\[ \tilde{I}_6 = (0.000,200.000,400.000) \]
\[ \tilde{I}_7 = (1.0,25,1.75) \]
\[ \tilde{I}_8 = (100.000,75.000,125.000) \]
\[ \tilde{I}_9 = (1.0,5.1.5) \]
\[ \tilde{I}_{10} = (1.0,75,1.25) \]
\[ \tilde{I}_{11} = (1.0,30,1.70) \]
\[ \tilde{I}_{12} = (1.0,40,1.60) \]

\[ \tilde{g} = (900.000,800.000,950.000) \]
\[ t_1 = (40.000,35.000,50.000) \]
\[ t_2 = (20.000,10.000,30.000) \]
\[ t_3 = (30.000,20.000,40.000) \]
\[ t_4 = (50.000,40.000,60.000) \]
\[ t_5 = (30.000,10.000,50.000) \]
\[ b_1 + t_1 (1- \alpha) = [800.000 + 40.000 \quad (1- \alpha)] \]
\[ 700.000 + 35.000 (1- \alpha) \]
\[ 900.000 + 50.000 (1- \alpha) \]

\[ t_2 + t_2 (1- \alpha) = [500.000 + 20.000 (1- \alpha)] \]
\[ 700.000 + 35.000 (1- \alpha) \]
\[ 100.000 + 50.000 (1- \alpha) \]
\[ 400.000 + 10.000 (1- \alpha) \]
\[ 600.000 + 30.000 (1- \alpha) \]

After these arrangements, we may choose two or three different auxiliary models to solve such as below:

Model-1

Max \( Z = \ldots \ldots \)

\[ X_1 + \ldots \leq 800.000 + 40.000 (1- \alpha) \]
\[ X_1 + \ldots \leq 500.000 + 20.000 (1- \alpha) \]
\[ X_1 + \ldots \leq 300.000 + 30.000 (1- \alpha) \]
\[ X_1 + \ldots \leq 100.000 + 50.000 (1- \alpha) \]
\[ X_1 + X_2 + X_3 + X_4 \leq 900.000 + 30.000 (1- \alpha) \]

Model -2

Max \( Z = \ldots \ldots \)

\[ X_1 + \ldots \leq 700.000 + 35.000 (1- \alpha) \]
\[ X_1 + \ldots \leq 400.000 + 10.000 (1- \alpha) \]
\[ X_1 + \ldots \leq 200.000 + 20.000 (1- \alpha) \]
\[ X_1 + \ldots \leq 75.000 + 40.000 (1- \alpha) \]
\[ X_1 + X_2 + X_3 + X_4 \leq 800.000 + 10.000 (1- \alpha) \]

Model -3

Max \( Z = \ldots \ldots \)

\[ X_1 + \ldots \leq 900.000 + 50.000 (1- \alpha) \]
\[ X_1 + \ldots \leq 600.000 + 30.000 (1- \alpha) \]
\[ X_1 + \ldots \leq 125.000 + 60.000 (1- \alpha) \]
\[ X_1 + X_2 + X_3 + X_4 \leq 950.000 + 50.000 (1- \alpha) \]

---
Solutions of these models with $\alpha = 0.80$ and their selected decision variables by algorithm listed below:

<table>
<thead>
<tr>
<th>Models</th>
<th>Solutions</th>
<th>Selected Var.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>$950.125,89 X_1$</td>
<td>$X_1$ and $X_3$</td>
</tr>
<tr>
<td>Fuzzy Mod.1</td>
<td>$1,065.023,24 X_1$</td>
<td>$X_1$ and $X_3$</td>
</tr>
<tr>
<td>Fuzzy Mod.2</td>
<td>$927.265,12 X_1$</td>
<td>$X_1$ and $X_3$</td>
</tr>
<tr>
<td>Fuzzy Mod.3</td>
<td>$1,187.317,64 X_1$</td>
<td>$X_1$ and $X_3$</td>
</tr>
</tbody>
</table>

4- Conclusion

As a result, we have seen from the above alternative solution list that maximum value of the objective function belongs to the 3rd Fuzzified Model. The approach has been given is based on the concept of a comparasion relation between fuzzy numbers. In application, top manegers always have the ability to improve their input capacities and system restrictions. In this application, only three alternative fuzzy model comparied by the original one. It ýs very obvious that there are unlimited version of the fuzzified alternatives. So that ýt is necessary powerfull software support to find out and criticies best solution between many logical models, representing original systems.

References