Abstract – This paper describes an approach to sensor bias modeling and estimation for ground target tracking applications using multiple airborne Ground Moving Target Indicator (GMTI) radar sensors. This approach was developed as part of the Precision Firecontrol Tracking (PFCT) segment of the DARPA Affordable Moving Surface Target Engagement (AMSTE) program. For airborne sensors, slowly varying platform location, heading and velocity errors lead to time-dependent measurement biases. Track accuracy can be improved by using a Kalman filter to estimate and correct the biases in real time, based on fixed reference points. The reference point location can be known a priori or estimated online as part of the bias correction algorithm. When the reference locations are known a prior, bias effects can be nearly completely eliminated. When the reference point is estimated online, significant performance improvement is obtained relative to uncorrected measurements.

Keywords: Bias correction, tracking, multi-sensor fusion.

1 Introduction

Currently, there is a great deal of interest in developing airborne radar systems to track moving ground targets such as tanks and trucks [1-5]. To separate targets from clutter, these systems use a type of Doppler filter call Ground Moving Target Indicator (GMTI) processing. GMTI systems are able to detect targets when their radial speed relative to the ground is greater than a system-specific Minimum Detectable Velocity, typically on the order of a few meters per second.

Airborne GMTI systems suffer from two distinct types of errors: random errors that are uncorrelated from one measurement to the next, and systematic or bias errors sources that vary slowly in time. The effect of bias error on system performance is quite different from that of random errors since it is not reduced by simple averaging. Potential sources of bias errors include sensor platform location, speed and heading error; atmospheric propagation error and range glint effects. Biases can affect all targets similarly, or their effects may vary by sensing region within the sensors field of regard, or they can vary from target to target. For example, sensor platform location, pointing, and velocity errors tend to affect all targets similarly. On the other hand, atmospheric propagation effects can change with bearing and elevation angle, effecting targets in different regions differently. Range glint (range error due to coherent interference from unresolved scatters) depends on target type and target aspect, and so tends to vary from target to target.

To estimate biases and correct them, reference points referred to as "fiducial targets" are required. This is simplest when the fiducial targets are surveyed fixed points. One complicating feature of GMTI is that target motion is required for detection. Surprisingly, GMTI returns can often be obtained from fixed objects with moving parts such as rotating antennas and pumps. These objects are referred to as "static movers". Thus, possible fiducial targets include surveyed static movers (static movers with known locations), non-surveyed static movers and non-static targets (movers) of opportunity.

When fiducial targets are available, their measurements can be input to a bias estimation filter. For fixed biases, this can be some kind of batch oriented least squares procedure. Often, the biases will not be fully observable, leading to ill-conditioning in the least squares problem which must be regularized. When biases are time varying, this procedure must be modified. One approach is to run the batch estimator periodically. In this case the estimates are independent from one batch process to the next.

In this paper we develop an extended Kalman filter (EKF) bias estimation algorithm as an alternative to batch methods. This EKF directly incorporates a bias dynamics model.

To deal with the problems of ambiguous report to track association and false alarms, most GMTI systems use some kind of Multiple Hypothesis Tracker (MHT) to process the raw reports provided by the radar signal processor. To support bias estimation, the MHT must identify fiducial targets. The fiducial targets and their associated measurements can then be used to estimate and correct the biases. From an estimation theoretic point of view, bias estimation and tracking would ideally form a single joint estimation process. However, this would be quite complicated to design. In this work, we chose to separate the tracker from the bias filter, although this undoubtedly imposes some (uncharacterized) loss in estimation performance.

This paper is organized as follows. Section 2 describes the measurement model for an airborne GMTI system, emphasizing the structure of the random and bias...
errors. Section 3 presents the bias dynamics model used here. Section 4 describes the MHT / bias filter system architecture and sketches how the measurement and dynamics models are incorporated into the bias filter. Section 5 shows the impact of bias estimation on the track accuracy in a typical example. Section 6 presents discussion and suggests a few directions for future work.

2 The Measurement Model

In GMTI applications, typical sensor to target ranges are on the order of 100 km or more, necessitating the use of global coordinate systems such as the WGS-84 system [6]. Its role in GMTI tracking and bias estimation is briefly synopsized here. Although GMTI sensors often provide radial velocity estimates, no radial velocity bias was assumed in the PFCT study and it is ignored here. Motion of each target and sensor is modeled in its local East-North-Up (ENU) frame, as shown in Figure 1. For ground targets the altitude is uniquely determined by the Digital Terrain Elevation Data (DTED) referenced to the geoid. With target latitude / longitude \((\varphi, \lambda)\), its 3D location is determined by its Latitude/Longitude/Altitude (LLA) \((l, j, A)\). The target velocity denoted \(y, x\) is restricted to lie in the ENU plane at \((l, j)\). The sensor reports its LLA location as \((s, s, A)\) (note that the sensor is airborne so \(s_A\) is not determined by the DTED).

The WGS-84 Earth-Centered Earth-Fixed (ECEF) coordinates are a fixed Cartesian frame useful for conversion between various ENU frames. Given LLA coordinates \((\varphi, \lambda, A)\), the corresponding ECEF location is \(X = (X, Y, Z)^T\) where

\[
Y = (N + A) \cos \varphi \sin \lambda \\
Z = ((1 - f)^2 N + A) \sin \varphi
\]

with \(N = a / \sqrt{1 - f(2 - f) \sin^2 \varphi}, \ a = 6378137\) m, \(f = 1/298.257223563\). (The parameters \(a\) and \(f\) define the WGS-84 ellipsoid.)

In the ECEF frame, the ENU unit vectors describing the tangent space at \((\varphi, \lambda)\) are

\[
M(\varphi, \lambda) = \begin{pmatrix}
-sin \lambda & -sin \varphi \cos \lambda & cos \varphi \cos \lambda \\
\cos \lambda & -sin \varphi \sin \lambda & cos \varphi \sin \lambda \\
0 & cos \varphi & sin \varphi
\end{pmatrix}
\]

\(\equiv \begin{pmatrix} e & n & u \end{pmatrix}\)  

(2)

In the ECEF frame, the vector from the sensor to the target is \(\hat{r} = x_t - x_s\). The range to the target is \(r = |\hat{r}|\). The sensor/target unit vector is \(\hat{r} = r / |r|\). The bearing \(\theta\) to the target (measured clockwise from \(n_s\)) is

\[
\theta = \arctan (\hat{r} \cdot n_s / \hat{r} \cdot e_s).
\]

(3)

In the sensor ENU frame, denote the target location as \(X_s = (x_s, y_s, z_s)^T\). In the absence of biases the sensor is at its reported location, which is at the origin in the sensor ENU frame. Then the range and bearing to the target are simply

\[
r = \sqrt{x_t^2 + y_t^2 + z_t^2}
\]

(4)

\[
\theta = \arctan(y_t / x_t)
\]

(5)

When biases and random errors are present, the platform location is offset from the reported location by the Cartesian vector \(X_b = (x_b, y_b, z_b)^T\). In addition, we assume that there is a range offset bias \(r_b\) and an azimuth bias \(\theta_b\) as well as random errors \(\delta r\) and \(\delta \theta\). The random component of the platform location error was assumed to be 0. With this, the complete expression in the absence of range-rate biases is given by

\[
r = \delta r + r_b + \sqrt{(x_t - x_b)^2 + (y_t - y_b)^2 + (z_t - z_b)^2}
\]

\(\theta = \delta \theta + \theta_b + \arctan((y_t - y_b) / (x_t - x_b))\).

(6)

(7)
The task of the bias filter is to estimate each of the time-varying quantities $x_b, y_b, z_b, \theta_b, r_b$ for each sensor. Initially, this may seem impossible without known reference points. There are two features of this problem that make bias estimation feasible, however. The first is that there are usually many targets and only a few sensors. Second, the biases are slowly varying in time. Thus each scan of data provides many measurements of the same set of bias parameters $x_b, y_b, z_b, \theta_b, r_b$ for each sensor.

### 3 Bias Dynamics Models

To develop a bias filter we must quantify the notion that the biases are slowly varying. The relevant time scale is set by the sensor sample rate. For typical applications this is between one measurement per second and one measurement per minute. The simplest model for a bias is just a fixed constant. The simplest non-constant model is characterized by the Ito equation

$$dx(t) = -\frac{1}{\tau} x(t) + dB(t)$$

with real $\tau > 0$ referred to as a time constant and $dB(t)$ is a scalar Brownian noise process with

$$E\{B^2(t-t')\} = q(t-t')$$

where the power spectral density $q$ is a constant. $x$ is a zero mean process with bounded variance

$$\sigma^2 = q\tau / 2.$$  \hfill (10)

One nice feature of this model is that, since it has bounded variance, bias estimates of components that are not observable still remain bounded. An example of how varying the time constant effects the trajectory of a Gauss-Markov process is shown in Figure 2.

### 4 MHT / Bias Estimation Architecture

The architectural relationship between the bias estimation filter and the MHT system is shown in Figure 3. The bias filter exploits the fact that the measurements $z$ are symmetric in the target state $X$, and the bias state $X_b$.

Where the target state in the target states to the measurements is given by Eqs. (6-7). Once the target states have been updated, the bias estimates for each sensor are estimated using an EKF that treats the target states $\hat{X}$, as known parameters.

To detail this process, let there be $S$ sensors. The bias state vector for sensor $s$, $s = 1, \ldots, S$ is $X_{bs}^s = (x_{bs}^s, y_{bs}^s, z_{bs}^s, r_{bs}^s, \theta_{bs}^s)$ where the coupling of the bias states to the measurements is given by Eqs. (6-7).

When the MHT receives a scan of data at time step $k$, it begins by time updating the state estimates for each track. The predicted track state is $\hat{X}_t^{k|k-1}$. To perform gating, association and measurement update. MHT must calculate the predicted measurement. The predicted measurement is determined by both the predicted target state and the predicted bias state $\hat{X}_t^{k|k-1}$ determined by the bias dynamics. Thus the MHT must use $z_t^{k|k-1} = h(\hat{X}_t^{k|k-1}, \hat{X}_s^{k|k-1})$ as its predicted measurement.

After the MHT processes each scan, it updates the bias estimate for the used sensor using the updated track estimates and measurements from the just-processed dwell. The first step is for the track measurement pairing module to select high confidence tracks (static movers, if available) and their associated measurements for use as fiducial points for bias estimation. Ambiguous tracks are excluded from the fiducial track list.

To perform the bias measurement update, let there be $F$ fiducial tracks. The track state estimates are $\hat{X}_f^{k+1}$, $f = 1, \ldots, F$ with measurements $z_f$, $f = 1, \ldots, F$. Then the bias measurement update is just an extended Kalman filter on the measurement vector...
with measurement model

\[ Z = \left( z^{T}_1, \ldots, z^{T}_F \right)^T \]  

(11)

To amplify on this notion, the measurements associated with the fiducial tracks are combined to form a single compound bias measurement and the state estimate for each track is treated as a known parameter in the measurement model for this compound measurement.

One subtle point is that the covariance of the fiducial tracks is required to evaluate the innovation covariance. This accounts for the fiducial point location uncertainty. In this way, well-located points such as stationary movers, beacons or road-constrained tracks receive greater weight in the bias estimate than less well-localized targets.

Once the bias estimation is completed, it is available for use by the measurement prediction module. MHT does not require direct access to the bias estimates. It only requires the predicted measurements \( z^{\hat{X}_1 \mid k} \) and their gradients \( \hat{H} \) to be computed using the bias estimate for the corresponding sensor. To accomplish this, measurement prediction requires knowledge of the predicted target state, the sensor, its reported platform state and the dwell time. Time update is performed on the biases to account for bias dynamics since the last bias estimate update. These are then used to calculate the predicted measurement and the measurement gradients. DTED corrections are applied at this point using the DTED altitude module.

### 5 Results

Figure 4 shows typical results obtained using the bias estimation and correction routines described above. These results are obtained using data from engineering data set 1 (EDS1) of the Defense Advance Projects Agency (DARPA) Precision Firecontrol Tracking (PFCT) program (for an overview, see [5]). These data were generated using centimeter accuracy vehicle tracks derived from Global Positioning System (GPS) data. Synthetic GMTI measurements were then generated from these GPS tracks, given sensor trajectories for a pair of sensors with a roughly 100 km stand-off range. The azimuth separation between the sensors was about 30°. For each sensor, the measurement update interval was 3 seconds. The standard deviations of the random error components are \( \sigma_r = 6 \text{ m in range, } \sigma_\theta = 2 \text{ mrad in azimuth and } \sigma_v = 2 \text{ m/s in range-rate.} \) In the figure, the

![Figure 3 – MHT / Bias processing architecture.](image_url)

When a scan of data is received, the outdated bias estimate is updated in the Bias and Measurement processing module using the bias dynamics model. The MHT predicted target state and the estimated bias are used in the measurement model to construct the predicted measurement for the MHT Kalman filters. The MHT automatically identifies static movers which are used as reference or fiducial tracks. The known or estimated track location and corresponding measurements are passed to the bias module where they are used to update the bias estimates for the corresponding sensor in preparation for the next scan.
The range bias standard deviation was the same set of biased measurements was used in each case. In each case, the same tracker was used with different methods of accounting for the biases. The same set of biased measurements was used in each case. The range bias standard deviation was \( \sigma_{r^b} = 6 \) m and the range bias time constant was \( \tau_r = 500 \) s. The azimuth bias standard deviation was \( \sigma_{\theta^b} = 2 \) mrad with time constant \( \tau_{\theta} = 300 \) s. The 3D platform location error standard deviation \( \sigma_p^b = 4.3 \) m (this corresponds to a 2.5 m error each x, y, z component) with a time constant of \( \tau_p = 120 \) s. No range-rate bias was applied to the measurements.

For the curve labeled “bias comp”, no attempt was made to estimate the bias. Instead, the measurement standard deviation in range and azimuth was inflated to compensate for the added uncertainty due to the biases. As the figure shows, this method is relatively ineffective. The RMS HTLE increases 4-fold to an average value of around 8 m. The curve labeled “bias est surveyed”, the bias is estimated and corrected using the bias Kalman filter with 5 surveyed static movers within the surveillance region. In this case, the bias filter was able to essentially completely eliminate the effect of the bias.

The curve labeled “bias est non-surveyed”, the same 5 static movers were used but no a priori information concerning their location was provided. In this case the non-surveyed static movers enable significant reduction in the target HTLE to about 5 m RMS. Finally, the curve labeled “bias est tracks” uses bias estimation based only on the moving targets within the surveillance region. In this case, there were a total of 3 targets within a roughly 100 square km region. In this case, no statistically significant reduction in the HTLE was obtained.

6 Discussion

We have shown that significant improvement in absolute track accuracy can be obtained for ground targets using an Extended Kalman Filter and surveyed or non-surveyed static movers to estimate time varying biases in range, bearing and platform location. It is not too surprising that this is effective when reference points with known static mover locations are available within the surveillance region, although it is interesting that the effect of the bias can be essentially completely eliminated with realistic target motion and sensor accuracy. What is perhaps somewhat surprising is that even when the reference point locations are not known a priori, significant reduction in error can be achieved. This is due to the averaging effect of the dynamic bias models used here. It is likely that greater reduction in error could be achieved over longer scenarios, but this has not been tested here.

It is also interesting that little improvement was obtained when only moving targets were used to estimate the biases. This should not be construed to mean that bias estimation has no utility in the absence of static reference points, however. In this case, there were only 3 targets in the surveillance region while in real GMTI applications, there are usually 10’s to 100’s of targets within the surveillance region. Furthermore, while the absolute error was not reduced, possible reduction in the relative error may have a significant positive effect in data association accuracy in multi-sensor applications.

While the separation of the bias estimation and track state estimation steps simplifies the implementation of the bias filter, it does lead to a theoretical loss in performance for the cases where no surveyed static movers are available. The reason for this is that the estimation error for the biases that the track states are correlated. This correlation effect seems to be significant but is not modeled here. To model this requires a fully coupled track and bias filter, greatly increasing the complexity of the algorithm. This awaits future investigation.

Acknowledgement

The authors benefited from discussions with J. Gleason and D. Klamer in this effort. This work was supported by the Defense Advance Projects Agency (DARPA) under the Affordable Moving Surface Target Engagement (AMSTE) program.

References


Figure 4 – Horizontal Track Location Error for a typical ground target using 2 airborne GMTI sensors. The two lower curves show similar performance is obtained using unbiased measurements or biased measurements with bias estimation and removal based on surveyed static movers (about 2 m RMS HTLE). When only non-surveyed static movers are available to estimate the biases, the RMS HTLE is about 5 m. When the bias estimate is based only on the 3 moving targets in this test scenario (solid curve), then the bias estimation algorithm described in the text provides little reduction in HTLE compared to not estimating the bias (small dotted curve).