**Mobile Radar Bias Estimation Using Unknown Location Targets**

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**Abstract** - In target tracking systems using radars on moving platforms the locations of these platforms is available from GPS based estimates. However, these estimated locations are subject to errors that are, typically, stationary autocorrelated random processes, i.e., slowly varying biases. In situations where there are no known-location targets to estimate these biases, the next best recourse is to use targets of opportunity at fixed but unknown locations. It is shown that these biases can be estimated in such a scenario, i.e., they meet the complete observability condition. Following this, the achievable accuracy for a generic scenario is evaluated. It is shown that accurate georegistration can be obtained even with a small number of measurements.

**Keywords:** Tracking, radar registration, airborne radar, debiasing.

1 Introduction

The problem of registration of radars has been discussed in the literature, e.g., in [4, 3] for fixed location radars. A detailed historical perspective can be found in [2]. Most of the work decoupled the bias estimation and the target tracking. The simultaneous estimation of sensor location bias and target tracking (but no sensor bias) for sensors whose location was modeled as a random walk was investigated in [7]. Observability issues were discussed in [6]. More recently, [8] considered the problem for radars on moving platforms, also by decoupling the tracking of targets of opportunity and the estimation of the radar and platform biases.

In the approach presented here, the bias estimation is carried out simultaneously with the estimation of the locations of the targets of opportunity, i.e., it is done optimally. These targets of opportunity are assumed at fixed unknown locations.

Section 2 presents the formulation of the problem. The proof of the observability of the biases and the target states as well as the conditions under which this holds are discussed in Section 3. The numerical results from Section 4 quantify the “estimability” of the biases, by evaluating the achievable accuracy for the bias estimates relative to the measurement noise in a generic GMTI (ground moving target indicator) radar scenario. Finally, Section 5 presents a discussion of the results and conclusions.

2 Formulation of the Problem

When multiple radars are used in the proper geometry, the final accuracy is determined primarily by each radar’s range estimation accuracy. In this case it is particularly important to extract the most accurate state estimate in the range direction. In particular, this requires reduction of the sensor biases, which are caused by sensor calibration and propagation uncertainties, as well as by navigation (platform location) uncertainties.

The true position of the platform is

\[ p(k) = \hat{p}(k) + \tilde{p}(k) \]  \hspace{1cm} (1)

where \( \hat{p} \) is the estimate of the platform position from the GPS navigation system and \( \tilde{p} \) is the corresponding error. This error is an autocorrelated process like, in general, the state estimation error coming from a Kalman filter (see, e.g., [1]). In addition, this is also due to the slowly varying propagation errors in the GPS system.

Consequently, the position error in (1) is modeled as the sum of slowly varying biases

\[ \tilde{p}(k) = \sum_{i=1}^{n_b} b_i(k) \]  \hspace{1cm} (2)

The dynamic model that will be used for the biases is

\[ b_i(k+1) = a_i b_i(k) + v_i(k) \quad i = 1, \ldots, n_b \]  \hspace{1cm} (3)

where, with \( T \) the sampling interval,

\[ a_i(k+1) = e^{-T/T_i} \]  \hspace{1cm} (4)
and \( T_i \) is the time constant of bias \( b_i \) in the range direction.

The process noises \( v_i(k) \) are zero mean, white and independent of each other. Assuming the stationary variance of \( b_i \) as \( \alpha_i^2 \), the variance of its process noise is easily seen to be

\[
q_i = \sigma_i^2 (1 - \alpha_i^2) \quad i = 1, \ldots, n_b
\]  

Other models, e.g., second order models can be also used.

The targets of opportunity are assumed at unknown but fixed locations

\[
x_j(k) = x_j \quad j = 1, \ldots, n_o
\]  

For the sake of simplicity of the presentation, all the variables to be estimated — the biases \( b_i \) and the target locations \( x_j \) — are assumed on the real line (the range direction). This will allow us to gain physical insight into the observability condition to be derived in the next section.

Without loss of generality we will assume that the estimated platform location is the origin — this causes only a known shift in the estimates and, consequently has no effect on the estimation accuracy.

The measurement equation for target \( j \) is

\[
z_j(k) = x_j - \sum_{i=1}^{n_b} b_i(k) + w_j(k) \quad j = 1, \ldots, n_o
\]  

with the standard assumptions about the measurement noises as zero-mean, white with covariances

\[
E[w_j(k)w_m(l)] = \sigma_w^2 \delta_{jm} \delta_{kl}
\]  

and independent of the process noises in the biases.

The composite state, of dimension \( n_b + n_o \), to be estimated is

\[
x(k) = \begin{bmatrix} b_1 \\ \vdots \\ b_{n_b} \\ x_1 \\ \vdots \\ x_{n_o} \end{bmatrix}
\]  

The state equation is

\[
x(k+1) = Fx(k) + v(k)
\]  

where

\[
F = \text{diag}[\alpha_1, \ldots, \alpha_{n_b}, 1, \ldots, 1]
\]  

and the covariance of the process noise is

\[
E[v(k)v(k)'] = Q = \text{diag}[q_1, \ldots, q_{n_b}, 0, \ldots, 0]
\]  

The measurement vector is

\[
z(k) = Hx(k) + w(k)
\]  

where

\[
H = [-I_{n_b \times n_b} \; I_{n_o \times n_o}]
\]  

where \( I_{n_b \times n_b} \) is the matrix of the indicated dimension with all elements unity, and \( I \) the identity matrix of the indicated dimension. The covariance of the measurement noise is

\[
E[w(k)w(k)'] = R = I_{n_o \times n_o} \sigma_w^2
\]

3 Observability of the State

It is shown next that the state (9) is completely observable from the measurements (13). The proof is given for \( n_b = 2 \) and \( n_o = 1 \). The extension to the general case is discussed later.

With the system matrix

\[
F = \text{diag}[\alpha_1, \alpha_2, 1]
\]  

and the measurement matrix

\[
H = [-1 \quad -1 \quad 1]
\]  

the observability matrix is

\[
O = \begin{bmatrix} H \\ HF \\ HF^2 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 1 \\ -\alpha_1 & -\alpha_2 & 1 \\ -\alpha_1^2 & -\alpha_2^2 & 1 \end{bmatrix}
\]  

which is clearly nonsingular as long as \( \alpha_1 \neq \alpha_2 \) and neither is 1. Note that a random walk bias model, which has \( \alpha = 1 \), is not observable in this case.

The full rank condition on the observability matrix guarantees that all the eigenvalues of the steady state covariance of the Kalman filter that estimates the state (9) are finite (see, e.g., [1], Sec. 5.2.5), i.e., the entire state can be estimated.

If \( \alpha_1 = \alpha_2 \neq 1 \), then the sum \( b = b_1 + b_2 \) is still observable. Note that this sum \( b \), the total bias, is what one is interested in. This will be evaluated numerically in the next section.

The above immediately generalizes to the case of arbitrary \( n_b \), with the condition being that the coefficients \( \alpha_i \) (which are the bias evolution eigenvalues in discrete time, determined by their time constants) are all different and none is 1 (which is the eigenvalue of the target’s position, which is fixed). Now, since all the biases and one target are observable from measuring that target’s position, the system consisting of \( n_o \) targets and biases is also completely observable from the \( n_o \) position measurements of these targets under the same condition as above.
4 Numerical Examples

Consider the case of \( n_b = 2 \) biases with time constants \( T_1 = 600s, T_2 = 120s \), and stationary RMS values \( \sigma_1 = 2 \) and \( \sigma_2 = 1 \). The number of opportunity targets is assumed as \( n_o = 5 \). The measurement noise variance is \( \sigma_w^2 = 1 \). The initial covariance matrix for the 7-dimensional state is

\[
P(0) = \begin{bmatrix}
\sigma_1^2 & 0 & -1_{1 \times 5} \sigma_1^2 \\
0 & \sigma_2^2 & -1_{1 \times 5} \sigma_2^2 \\
-1_{5 \times 5} \sigma_1^2 & -1_{5 \times 5} \sigma_2^2 & I_{5 \times 5} \sigma_w^2 + 1_{5 \times 5}(\sigma_2^2 + \sigma_1^2)
\end{bmatrix}
\]

(19)

The results for the state estimation covariance \( P \) corresponding to the state (9), for sampling interval \( T = 1s \) are given in Table 1. These were obtained by iterating the Riccati equation of the optimal linear (Kalman) filter corresponding to (10) and (13) (see, e.g., [1]). Only the top 3\times3 block of the matrix is shown because the terms corresponding to each target are the same as that of the first one, which is \( P_{33} \).

The nonzero off-diagonal terms in the last column or row of \( P \) indicate the coupling between the bias estimation and the state estimation. Thus, any approach that ignores this coupling, is not optimal.

Note that the variances of individual bias estimates seem fairly large — comparable with the measurement noise variance \( \sigma_w^2 \). However, what matters is the total residual bias, whose variance is, based on the elements of the covariance matrix \( P \), given by

\[
\sigma_b^2 = E[(\tilde{b}_1 + \tilde{b}_2)^2] = P_{11} + P_{22} + 2P_{12}
\]

(20)

where \( \tilde{b}_i \) is the estimation error for bias \( i \). It can be seen that the total residual bias MS value is close to a tenth of the measurement noise variance \( \sigma_w^2 = 1 \) for the short sampling interval \( T = 1s \). Thus, in this case, the residual bias amounts to about 7% of the measurement error standard deviation.

Table 2 shows the residual bias variance as a function of the sampling interval. Note that even for the higher sampling interval (lowest revisit rate), the residual bias amounts only to about 17% of the measurement error standard deviation.

5 Discussion of the Results and Conclusion

Registration of moving radars — estimation of their platform location bias and sensor bias — can be accomplished using targets of opportunity at fixed but unknown locations when the biases have non-unity eigenvalues in their noise-driven discrete time dynamic models. It is shown that accurate georegistration can be
obtained even with a small number of measurements. The residual bias can be reduced to a small fraction of the measurement noise standard deviation.

References


