Implementation of Hough Transform as Track Detector

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Abstract. Hough Transform is a convenient tool for features extraction from images. In this paper an implementation of Hough Transform is considered for automatic track initiation in the surveillance radar space. The need of track initiation arises when there are many moving objects in the sensor surveillance volume and when it is not clear which measurement to which target belongs. If the type of target trajectories is known, the trajectories can be easily detected by performing corresponding Hough transform on the track image. Here, the effectiveness of the Hough transform track initiator is discussed. The influence of Hough parameter space granularity upon probability of track detection is analyzed. Analytical expressions for probability of track initiation using Hough Transform are derived in the presence of normal distributed additive system noise, measurement noise and without any noises. A new parameter space structure, matching with measurement errors is proposed. The Monte Carlo simulation confirms received analytical result.

Keywords: Hough Transform, Track Initiation

1. Introduction

The identification of dynamic objects can be described as a task consisting of the following three different sub problems - object’s state identification, model identification and measurement identification.

In this paper the problem of interest is the measurement identification. Some times this problem is referred to as measurement association. The problem arises when there are many moving objects in the sensor surveillance volume and when it is not clear which measurement to which target belongs. The situation is complicated additionally by extra measurements, generated by clutter or noise in the radar receiver. The solution of described problem finds right affiliation of every measurement.

The measurement identification problem increases the complexity of the general identification task. This task can be solved in two steps: At the first step the measurement identification problem is solved. The state estimation is considered at the second step. The consecutive solution of common task is feasible when the measurement errors are relatively small in comparison with the distances between objects. In the conversely case it is obligatory to do state estimation of the whole set of possible combinations between radar measurements and potential targets. So, in this case the number of hypothesis grows exponentially with the number of measurements.

A convenient tool for measurement association into trajectories and for reducing the number of hypothesis is Hough Transform (HT). HT is well-known technique in image processing [4]. A search radar detection and track with HT was described by Carlson, Evans and Wilson in an excellent and awarded series of three papers [5, 6, 7]. The authors received detection statistics of HT track initiator. They found that a very fine granularity of HT parameter space would split a target into several parameter sells while a coarse granularity would force the noise to add up in fewer and fewer cells. But in these papers the authors did not examine losses of HT as track detector. As a result of the parameter space granularity, the parameters even of the nearest accumulator could not always coincide with the trajectory parameters. Thus, for an arbitrary chosen trajectory it could not be guaranteed that all of its points would fall in a single accumulator. Because of the system and measurement errors, the votes can be additionally dispersed in several accumulators. If the accumulator size is too small (with respect to error deviation) the peak can be scattered in several accumulators and track couldn’t be detected. Therefore, the choice of suitable accumulator size and track detection threshold is a problem of big importance.

In the present paper the losses of HT track detector are investigated. HT is used to detect rectilinear target tracks in a two-dimensional radar data space. The 2D data space is used since the concepts are more clearly described and explained in this case. The transition to the 3D data space can be regarded as two consecutive 2D tasks. The influence of parameter space granularity upon the probability of track detection of HT track initiator is analyzed. Three cases are considered: without any noises; in the presence of system noise only and in the presence of system and measurement noises. In the last case a new structure of the HT parameter space is proposed.

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2. Hough Transform

HT was patented in 1962 for straight line detection in pictures [1]. The original transformation equation, based on slope-intercept parametrization of lines, was improved by Duda and Hart [2] through the use of angle-radius parametrization:

\[ \rho = x \cos \theta + y \sin \theta \]

Their procedure is based on the normal parametrization of a line. Such a parametrization specifies a straight line by its distance \( \rho \) from the origin and the angle \( \theta \) of its normal. Computational efficiency is obtained by specifying an acceptable tolerance in \( \rho \) and \( \theta \) and quantizing the \( \rho - \theta \) plane into a quadruled grid. The line detection procedure is implemented by considering the quadruled grid as a two-dimensional histogram or array of accumulators. The procedure is illustrated in Fig. 1 where the original picture (three points \( L_1, L_2 \) and \( L_3 \) lying on a straight line a) is displayed.

![Figure 1: Normal straight line parametrization](image)

In Fig. 2 two-dimensional histogram is displayed by projecting all three picture points to the \( \rho \)-axis of each \( \theta \) value. The number of points that fall in each cell shapes the height of histogram. To complete the line detection procedure a suitable threshold has to be defined. If the count in a particular cell in the \( \rho - \theta \) histogram exceeds the threshold, a line is detected and specified by the corresponding values \( \rho \) and \( \theta \) of this cell.

![Figure 2: Vote histogram in Hough parameter space](image)

3. HT track initiator

The problem of track initiation arises when sensor observes many objects and received measurements have uncertain origin or when there are measurements generated by noise. In this case HT can be applied as a robust and effective method for finding lines fitting a set of 2D points [4]. The HT has a number of interesting properties:

- HT can be applied for detection of several simultaneously existing lines. The measurements from every one of them form local maximum in parameter space.
- HT is very robust to the noise produced by isolated noisy measurements and missed measurements (since their votes do not affect significantly the local maxima).
- The parameters of local maxima in the Hough domain are good approximation for state estimation algorithms.

The HT was generalized to detect features with arbitrary shapes [3], but in this paper it is used only for straight line detection. The rectilinear trajectory is regarded as universally accepted model used for track initiation. It is because the straight line motion dominates in object motion. We assume also that the targets in the surveillance volume are moving with constant speed.

4. Track initiation without noises

When system noise and measurement noise absent the estimated system can be described by following state and measurement equations (under consideration that sensor works in 2D polar coordinate system):

\[
\begin{align*}
    x(k+1) &= F(k)x(k) \\
    z(k+1) &= H[x(k+1)]
\end{align*}
\]

where the state vector and transition matrix are:

\[
x = \begin{bmatrix} x \\ y \end{bmatrix}, \quad F(k) = \begin{bmatrix} 1 & 0 & T \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad T \text{ is sampling interval.}
\]

The polar position measurements are:

\[
z(k+1) = \begin{bmatrix} r \\ \alpha \end{bmatrix}, \quad r = \sqrt{x^2 + y^2} \quad \text{and} \quad \alpha = \arctan\left(\frac{y}{x}\right).
\]

Let denote the target speed by \( v = \sqrt{x^2 + y^2} \) and track duration by \( n \) (the number of sampling intervals with time length \( T \)). It is obvious that all measurements from such a target will lie on a rectilinear trajectory segment with length \( L = v(n-1)T \).

Let suppose that the chosen accumulator size is \((\Delta \rho, \Delta \theta)\). A rectangular stripe with width \( \Delta \rho \)
corresponds to every one accumulator \((\rho, \theta_i)\) in the surveillance volume. The stripe is placed on distance \(\rho\) at angle \(\theta_i\). If a measurement lies in this stripe its vote takes part in the histogram of this accumulator. For a given \(\theta_i\), the accumulators \((\rho, \theta_i), \ldots, (\rho, \theta_i)\) conform to \(N_{\rho}\) parallel stripes in the measurement space. Let suppose that the parameters of a random trajectory are \((\rho, \theta_i)\). It is obvious that there is a set of accumulators \(\zeta = \{(\rho, \theta_i), i = 1, \ldots, N_{\rho}\}\), for which \(|\theta_i - \theta_0| \leq \Delta \theta/2\). Let denote the middle point of the trajectory by \(L_m\). The point \(L_m\) lies in one of the stripes, corresponding to accumulators in the set \(\zeta\). We call this stripe the nearest to the trajectory and denote corresponding accumulator by \((\rho, \theta_i)\).

We are interested in probability \(P_G\) that a trajectory with length \(L\) will not be split between several stripes. This problem is very similar to Buffon type problem. The well known Buffon needle problem was posed by its author in 1733 at a meeting of Academy des Sciences de Paris. The problem sounds like that [8]: in a room, the floor of which is merely divided by parallel lines, at a distance \(a\) apart, a needle of length \(l < a\) is allowed to fall at random. Which is the probability that the needle intersects one of the lines? The solution, determined by Buffon by means of empirical methods, was \(P = \frac{2l}{\pi a}\) [8].

In 1812, Laplace extended the problem by considering a room paved with equal tiles, shaped as rectangles of sides \(a\) and \(b\), with \(l < \text{min}(a,b)\). The solution was \(P = \frac{2l(a + b) - l^2}{\pi ab}\), and it is obvious that the probability of Buffon can be obtained from that of Laplace by letting \(b \rightarrow +\infty\).

Let denote \(b_i = \arcsin \left(\frac{\Delta \rho}{L}\right)\). The probability \(P_G\) that a trajectory with length \(L\) does not intersect any set \(\zeta\) of parallel lines can be expressed as follows:

\[
P_G = \frac{2 \int_0^{\min(0.5a, b_i)} \int_{-0.5\rho}^{0.5\rho} \frac{d\varphi}{\rho} \frac{d\rho}{0.5\rho} \rho \cos(0.5\rho) \sin \varphi} = \frac{\Delta \theta \Delta \rho + 2L \cos(0.5\Delta \theta) - 1}{\Delta \theta \Delta \rho} \quad \text{when} \quad 0.5\Delta \theta \leq b_i
\]

\[
= \frac{2b_i \Delta \rho + 2L (\cos b_i - 1)}{\Delta \theta \Delta \rho} \quad \text{when} \quad 0.5\Delta \theta > b_i
\]

5. Track initiation in the presence of system noise

In the presence of system noise the estimated system is described as follows:

\[
x(k + 1) = F(k) x(k) + v(k)
\]

\[
z(k + 1) = H[x(k + 1)]
\]

The system noise \(v(k)\) is white and normal distributed with covariance matrix \(P\). Let consider that the system noise is composed by transversal and longitudinal independent normal distributed constitutions with corresponding standard deviations \(\sigma_t\) and \(\sigma_z\). The ideal target position is corrupted by system noise to real target position, which parameters are measured without errors. Let a trajectory is determined by \(N\) measurements. Now we define the event \(A(i)\) that \(i\)-th measurement from the trajectory falls in the nearest stripe. Let denote the probability of this event by \(P_i\). Then, the probability \(P_G\) can be expressed as:

\[
P_G = \prod_{i=1}^{N} P_i
\]

The ideal target position at the chosen moment can be expressed as a function of the trajectory parameters. It is important to express only the target distance to the stripe borders (because of unlimited stripe length). Let denote the distance between trajectory center \(L_m\) and one of the stripe borders by \(d_m\). The ideal target position in the stripe at the \(i\)-th measurement moment is given by expression:

\[
d_i = d_m - \left[\frac{L}{2} - (i - 1)v_T\right] \sin(\theta_i - \theta_n)
\]

The probability of \(A(i)\) is defined by this part of pdf of system noise, which lies in the nearest stripe:

\[
P_i(\theta_i - \theta_n, d_i) = \frac{1}{2\pi\sigma_t\sigma_z} \int_{y_1}^{y_2} \frac{1}{\sigma_t^2 + \sigma_z^2} \frac{y_1^2}{\sigma_t^2} \frac{dy_1}{\sigma_z^2} d\varphi
\]

where \(a_1 = a_2 = \tan \left[\pi/2 - (\theta_i - \theta_n)\right]\), \(b_1 = \frac{d_i}{\sin(\theta_i - \theta_n)}\), and \(b_2 = \Delta \rho - \frac{d_i}{\sin(\theta_i - \theta_n)}\).

The probability \(P_G\) is written as:

\[
P_G = \frac{\prod_{i=1}^{N} P_i(\varphi, r) dr d\varphi}{\Delta \theta \Delta \rho}
\]
6. Track initiation in the presence of measurement errors

In the presence of measurement errors ($\delta r \sim N(0, \sigma_r)$ and $\delta \alpha \sim N(0, \sigma_\alpha)$) are mutually independent) the estimated system can be described as follows:

$$
\begin{align*}
    x(k+1) &= F(k)x(k) + v(k) \\
    z(k+1) &= H[x(k)] + w(k)
\end{align*}
$$

where the covariance matrix of measurement errors is:

$$
R = \begin{pmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\alpha^2 \end{pmatrix}.
$$

It is not so straightforward to calculate $P_G$ as the cases above. That is because the equal probability gates of measurement errors (Fig.3) have different sizes as a function of distance from the sensor. To overcome this drawback a two level structure of HT parameter space is proposed - fine level and coarse granularity level. The fine granularity level cells have smaller size than coarse granularity level ones. Let denote the size of fine granularity level cells by $(\delta \rho, \delta \theta)$ and the size of coarse granularity level cells by $(\Delta \rho, \Delta \theta)$. The size of coarse granularity level cells is identical to accumulator size in the explanation above and coarse cells act as accumulators. The size of smaller cells is chosen as follows: $\delta \rho = \Delta \rho$ and $\delta \theta = \frac{\Delta \theta}{k_\theta}$, where $k_\theta$ is a positive integer.

To be covered area of the complex stripe entirely an additional condition is introduced on the number of elementary cells in the accumulator:

$$
\delta \theta = \frac{\Delta \theta}{k_\theta} \leq \arccos \frac{\rho}{r_{\max}} - \arccos \frac{\rho + \Delta \rho}{r_{\max}}.
$$

The worst case is when $\rho = 0$. In this case inequality transforms to:

$$
k_\theta \geq \frac{\Delta \theta}{\pi} - \arccos \frac{\Delta \rho}{r_{\max}}.
$$

The distance error will be compensated by suitable stripe width (Fig.6). As it could not be compensated exactly the strip width is chosen in the worst case. To form gate with size $\pm k_r \sigma_r$ it is necessary to choose accumulator size $\Delta \theta = 2k_\alpha \sigma_\alpha$.

Now we try to compensate the azimuth error $\varepsilon_\alpha$ with such type of complex stripe. Let the target moves on the straight line $l$ (Fig.5) on the distance to the sensor $\rho$. It can be proven that lines of equal probability of azimuth error are the tangents through points $E$ and $C$ $(EF$ and $CD)$. The points $E$ and $C$ lie on the circle with radius $\rho$ and angles $EOB = BOC = \varepsilon_\alpha$. Hence, to form a gate with size $\pm k_\alpha \sigma_\alpha$ it is necessary to choose accumulator size $k_\theta = \frac{\Delta \theta}{\arccos \frac{\rho}{r_{\max}}}$.
The detection probability $P_D$ of Hough track detector can be expressed as a product of the two probabilities $P_G$ and $P_{D_{an}}$:

$$P_D = P_{D_{an}} P_G,$$

where $P_{D_{an}}$ denotes the probability that needed number of measurements (according to the chosen threshold) is detected. The expressions for $P_{D_{an}}$ is derived in the case of noncoherent integration in [6]:

$$P_{D_{an}} = 1 - (1 - P_D)^N - \sum_{m=1}^{\frac{N}{m}} \left[ \frac{N}{m} \right] p_D^m (1 - p_D)^{N-m} \times \left[ 1 - \ell \sum_{k=0}^{\frac{\xi-m\eta}{1+\eta}} \frac{\xi-m\eta}{k!} \right],$$

where $p_D = \frac{\eta}{1+\eta}$ is a primitive detection probability for a Swerling II type target model with Rayleigh distributed noise amplitude, $S$ is SNR, $\eta$ is threshold constant, $\xi$ is the threshold in the parameter space accumulator and $m^*$ is defined as: $m^* = \text{ceil}(\xi / \eta)$ [6].

In the binary integration case when the primitive $P_D$ are different an iterative approach called Brunner’s method is used [7]:

$$P_{D_{an}} = \sum_{m=0}^{N} P(m, N)$$

Here $P(m,n)$ is cumulative probability of getting exactly $m$ detections from $n$ looks at the target, $M$ is detection threshold, $P_D(k)$ is the primitive probability of detection from the $k$-th time slice and $P(0,n) = [1 - P_D(n)] P(0, n-1)$,

$$P(m,n) = [1 - P_D(n)] P(m,n-1) + P_D(n) P(m-1,n-1),$$

$$P(n,n) = P_D(n) P(n-1,n-1).$$

In the simplest case of binary integration and equal primitive $P_D$ the probability $P_{D_{an}}$ is expressed as follows:

$$P_{D_{an}} = \sum_{m=0}^{N} \left[ \frac{N}{m} \right] p_D^m (1 - p_D)^{N-m}.$$

8. Simulation results

A verification of received results is done by simulation. The first case describes a system without noises. The probability $P_G$ is calculated for different accumulator size $\Delta \rho = 0.125 \pm 3km$ and for different trajectory lengths 18 km, 30 km and 42 km (solid lines on Figure 7). The same probability is received by Monte Carlo simulation (10000 runs) - dashed lines on Figure 7. The coincidence is obvious.
size $\Delta \rho = 0.125 \pm 3km$ and trajectory lengths $10km$, $20km$ and $30km$). The solid lines correspond to results, computed by analytical expression, and the dashed lines are received by Monte Carlo simulation (50000 runs).

The last simulation (Fig.9) concerns a system in the presence of system and measurement noise. The results are analogous to the previous case, but the values of $P_G$ from Monte Carlo simulation are higher then the results from analytical expression. The main reason of this disagreement is not exact distance error compensation (the worst case).

9. Conclusions

In the present paper detection characteristics of HT track initiator are discussed. The influence of Hough parameter space granularity upon probability of track detection is analyzed. The corresponding analytical expressions for probability of track initiation are derived in the cases without any noises, in the presence of system noise only and in the presence of measurement and system noises. An extensive Monte Carlo simulation confirms the received results.

10. References