On parametric detection of small targets in sea clutter

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Abstract - A new algorithm is presented that instantaneously estimates the clutter characteristics in the environment of the radar cell that is processed. No a priori information is used, since the algorithm operates directly on the data of the incoming burst. It is a technique that adapts continuously and instantaneously to the environment. The algorithm, after having identified the sea clutter, rejects this sea clutter from the data; thus enhancing the probability of detecting small objects in a clutter environment. The final step is the actual detection that makes use of the advantages of the parametric representation. This results in a lower rate of false detections and as a consequence the later stages, like clustering and tracking, receive a more accurate input. The technique behind this algorithm uses recent developments in parametric time series analysis and performs well in suppressing sea as well as land clutter.

Keywords: clutter, automatic clutter identification, automatic clutter rejection, parametric filtering, time series analysis.

1 Introduction

The operational demands change for defence radar systems, since the need arises to detect small targets in environments dominated by sea clutter. These demands are not only related to littoral warfare situations, but also apply to tasks like coastal surveillance. To meet the changing functional demands, improved signal processing algorithms are needed.

Signal processing algorithms estimate information from a radar signal. The goal is the detection and -if possible- the identification of targets in an environment dominated by clutter. Clutter is regarded as unwanted echo signals from the environment, that should preferably be suppressed in order to reach a sufficiently low rate of false alarms. The natural constraint is of course that even a high resolution signal processing algorithm cannot extract more information than is contained in the signal.

The processing, proposed in this paper, is based on parametric estimation methods and uses autoregressive models for Doppler spectral estimation. It is pointed out how a reliable model of sea clutter can be constructed, for the identification of sea clutter. The parametric representation is also used for high resolution Doppler detection. Since the Doppler spectra are estimated on the basis of only a small number of radar returns, recent developments in time series analysis are used [2][3][4][13], especially the treatment of relative short series. It follows that the statistics of the estimates, like bias, variance, etc., depend to a great extend on the estimation method used.

The parametric description of the filtered Doppler spectrum allows for advanced detection algorithms. For instance, the peak frequencies can be calculated analytically. This will lead to more accurate detections and a lower probability of false alarm on clutter.

2 Radar sea clutter

Radar clutter arises in several forms. In this paper the emphasis has been put on extended clutter with diverse Doppler characteristics. Examples are e.g. sea clutter and land clutter caused by trees and bushes. In the presence of these types of clutter it is particularly difficult to detect slowly moving objects with a small RCS. They literally “drown” in the echoes of the clutter environment. Still one may want to detect small boats or floating objects on the sea surface or slow moving objects surrounded by vegetation. The main characteristic of these situations is that the target as well as the clutter have a Doppler speed, which is in the same order of magnitude. The only difference is the clutter being extended on a number of range cells, while the target has a point representation.

Clutter can be identified by its Doppler characteristics. Haykin [8] has worked on the identification of different types of clutter by means of autoregressive models. By analyzing the positions of the reflection coefficient of AR models, several clutter types could be identified [7]. The values of the estimated reflection coefficients from sea clutter appear to be highly dependent on the meteorological conditions. Therefore results may differ drastically from realization to realization. Instead of identifying the clutter using averages of many realizations, the Doppler structure of the clutter is estimated based on the data of the burst at hand. Averaging adjacent range cells yields a clutter reference model, that is used to construct a filter that instantaneously rejects the clutter.
3 Parametric spectral estimation

The autoregressive model describes the data series $x_t$ as the response of an all-pole filter to white noise $e_t$. The AR($p$) model is given by:

$$x_t + a_1 x_{t-1} + a_2 x_{t-2} + ... + a_p x_{t-p} = e_t.$$  \hspace{1cm} (1)

Since the AR model describes the spectral peaks of stationary stochastic signals, it is well suited to radar Doppler processing. To infer the model on the data at hand the parameters $\hat{a}_i$ have to be estimated and the order $p$ has to be selected.

The residual variance $S^2(p)$ (see also (9)) describes the fit of the estimated model to the series from which it is estimated. The residual variance will always decrease when extra reflection coefficients are estimated. It will reach its lowest value for a model of order $n-1$.

The prediction error $PE(p)$ describes the fit of the model to new and independent data from the same process that generated the series. The $PE(p)$ has a minimum for the model configuration (parameters and model order) that gives the optimal representation of the data. For higher model orders the $PE(p)$ will increase. Order selection criteria (see section 3.2) generally select the model that has smallest $PE(p)$ in expectation [13]. A small Prediction Error in the time domain means at the same time an optimal fit in the frequency domain [10]; that means no spurious peaks in the spectrum. When the process is known, the prediction error is calculated by:

$$PE(p) = \hat{a}^T R \hat{a},$$ \hspace{1cm} (2)

where $R$ is the covariance matrix of the original process and $\hat{a}$ are the estimated parameters. Here the results of estimation, represented by $\hat{a}$, are compared to the original process, represented by $R$. This exercise can of course only be carried out in simulation experiments, where the original process generating the synthetic data is known. As such the $PE(p)$ is a powerful tool in calibrating algorithms and in comparing the quality of competing descriptions. In this paper the $PE(p)$ will be used to that extend in section 4.

3.1 Parameter estimation

The best method to estimate AR parameters from a given series, is the Burg method [9]. The Burg method estimates reflection coefficients $k_i$ in a recursive way, which ensures the algorithm to estimate a stationary model; all estimated poles are inside the unity circle [5]. Another property of this method is that an unbiased estimate of the autocovariance function can easily be inferred from the estimated $k_i$. The importance of this characteristic is pointed out in [1].

The Burg method estimates a first order model from a given series, using

$$\hat{k}_1 = -R_1/R_0.$$ \hspace{1cm} (3)

The unbiased estimate of the autocovariance function can be used, since $R_0$ is always larger than $R_1$, which guarantees a stationary model. Then, the series is filtered forward and backward by the estimated reflection coefficient, resulting in two residual series:

$$\hat{e}_{f,k} = x_t + \sum_{i=1}^{k} \hat{a}_i x_{t-i}$$

$$\hat{e}_{b,k} = x_{t-k} + \sum_{i=1}^{k} \hat{a}^*_i x_{t-k+i}$$ \hspace{1cm} (4)

where $\hat{a}^*$ denotes the complex conjugate of $a$. From these residuals, the next reflection coefficient is estimated:

$$\hat{k}_k = \frac{-2 \sum_{i=1}^{k} \hat{e}_{f,i-k} \hat{e}_{b,i-k}^*}{\sum_{i=1}^{k} (|\hat{e}_{f,i-k}|^2 + |\hat{e}_{b,i-k}|^2)}.$$ \hspace{1cm} (5)

Since

$$\frac{2 \cdot a \cdot b}{a^2 + b^2} \leq 1,$$

the new reflection coefficient estimate will always be inside the unit circle, and the model will always be stationary.

The next order residuals are calculated by:

$$\hat{e}_{f,k+1} = e_{f,k} + \hat{k}_k \hat{e}_{b,k}$$

$$\hat{e}_{b,k+1} = e_{b,k} + \hat{k}_k^* \hat{e}_{f,k}.$$ \hspace{1cm} (6)

The Levinson-Durbin recursion is used to calculate parameters from the estimated reflection coefficients:

$$\begin{bmatrix}
\hat{a}_{i,j+1} \\
\vdots \\
\hat{a}_{i+1,j+1}
\end{bmatrix} = 
\begin{bmatrix}
\hat{a}_{i,j} \\
\vdots \\
\hat{a}_{i+1,j}
\end{bmatrix} + 
\hat{k}_{i+1} \begin{bmatrix}
\hat{a}_{i,j}^* \\
\vdots \\
\hat{a}_{i+1,j}^*
\end{bmatrix},$$ \hspace{1cm} (8)

where $\hat{a}_{i,j}$ is the $j$-th parameter of the $i$-th order AR model. As shown in this expression, the AR model of order $i+1$ is calculated from the AR model of order $i$ and the reflection coefficient $k_{i+1}$.  

3.2 Order selection

Order selection is an important and much studied problem in the field of time series analysis. The problem becomes even more difficult to handle, when only a small number of observations is available. This is the case in radar Doppler processing, especially if the algorithm has to act directly on the data of one burst. Solutions have been found thanks to recent advances in the field of finite sample theory, [2][3][4][13].

Of all estimated autoregressive models, from order 0 up to order $n-2$, order selection should select the model order $p$ that gives the best fit of the model to the process that generated the data. When the order is chosen too high, not only the process is described, but also some statistical fluctuations of this particular realization. This situation is called overfit and leads to spurious spectral peaks.
However, when the order is chosen too low, not all details of the process are described. This is called underfit. The risk between overfit and underfit has to be balanced by the order selection criterion \([3][12][13]\).

Order selection criteria use the residual variance \(S^2(p)\), which indicates the fit of the model of order \(p\) to the data that it is estimated from. The residual variance of an AR\((p)\) model can be calculated by:

\[
S^2(p) = \sigma^2_x \sum_{i=1}^{p} \left(1 - |k_i|^2\right),
\]

(9)

in which \(\sigma^2_x\) is the variance of \(x\). The residual variance will always decrease when extra reflection coefficients are taken into account.

Furthermore, the expectation of the variance of reflection coefficients, estimated from white noise, is used in order selection. Asymptotically, when estimating reflection coefficients from white noise, their variance equals \(1/n\), where \(n\) is the number of observations. When estimating from a finite, or small, number of observations of white noise, the reflection coefficient variance is no longer equal to \(1/n\). For the Burg method, the finite sample variance depends on the estimated order \(i\) and the number of observations \(n\). It can be described by the finite sample variance coefficient, \([2]\),

\[
v(i, \text{Burg}) = \frac{1}{n-i+1}.
\]

(10)

The finite sample coefficient \((10)\) depends on the method used to estimate the autoregressive parameters, the number of observations and the order of the estimated reflection coefficient. When selecting the best model order, these finite sample variances should be taken into account. Order selection criteria that do, are the FIC, or Finite Information Criterion, \([3]\),

\[
FIC(p, \alpha) = \log(S^2(p)) + \alpha \cdot \sum_{i=0}^{p} v(i, \cdot),
\]

(11)

and the FSIC, or Finite Sample Information Criterion, \([4]\),

\[
FSIC(p) = \log(S^2(p)) + \prod_{i=0}^{p} \frac{1 + v(i, \cdot)}{1 - v(i, \cdot)} - 1.
\]

(12)

Advantages of both methods can be combined to the CIC, Combined Information Criterion, \([1]\),

\[
CIC(p) = \max\{FSIC, FIC\}.
\]

(13)

The order for which CIC has its minimum is the order that has the best expected model fit and yields no spurious peaks in the spectrum; the Doppler spectrum can now be calculated by:

\[
\hat{S}_{xx}(\omega) = \frac{\sigma^2_x}{2\pi} \frac{1}{|1 + \hat{\alpha}_1 e^{-j\omega} + \ldots + \hat{\alpha}_p e^{-j\omega\tau}|^2}.
\]

(14)

## 4 Averaging adjacent range cells

### 4.1 Problem definition

The problem addressed in this section is to find the best way to estimate an average model from multiple realizations of the same stochastic process. A good technique is needed, since a number of range cells, say \(m\), in the environment of the cell that will be filtered, have to be averaged in order to construct the instantaneous clutter filter.

Suppose the data in the \(m\) adjacent range cells is generated by the same stochastic process. The information in these \(m\) cells, each containing \(n\) observations, has to be averaged in order to come up with one average AR model, which represents the mean-information in these cells. Three candidates are distinguished, that can be used to average the information in the adjacent cells: the parameters, the reflection coefficients and the autocovariance functions. Averaging of different representations will lead to different characteristics of the final model. The averaging procedure to be chosen must satisfy the property that averaging stationary models results again in a stationary model. Of those methods satisfying this demand, the method having the lowest prediction error will be chosen, since this method gives the best model fit, and therefore the best spectral estimate.

First a theoretical analysis will be given. Then these results will be illustrated with a simulation.

### 4.2 Theoretical analysis

Parameter vectors represent a stationary model, if, in the \(z\)-domain, their roots are all inside the unit circle. This group of parameter vectors does not form a convex space, which means that it is possible that, although all individual parameter vectors represent stationary models, their average does not.

Reflection coefficients and autocovariances from stationary models, do form convex spaces. This means that their average also represents a stationary model and therefore the representations can be averaged.

Averaging autocovariance functions will always lead to an autocovariance function. These functions, which have a non-negative spectrum, represent a stationary model, if the autocovariance function is a positive semi-definite function. The sum of two positive semi-definite functions can be proved to be also positive semi-definite. This can be seen from the fact that the positive semi-definite property of a function implies that its Fourier transform is real and non-negative, and vice versa. Because the spectrum is a direct Fourier transform of the autocovariance function and because a summation in the time domain equals a summation in the frequency domain, averaging two positive semi-definite autocovariance functions is the same as averaging two non-negative spectra. This will lead to a non-negative (average) spectrum, and so to a positive semi-definite autocovariance function.

Both reflection coefficients and autocovariance functions are in principle suited for averaging. The prediction error, given in expression \((2)\), will be used to discriminate between these two possibilities. A theoretical
analysis will be given here for the AR(1) model. In this case, the prediction error simplifies to:

\[
PE_{AR(1)} = R_0 + R_0\hat{a}^2 + 2R_1\hat{a} = R_0 + R_0\frac{\hat{R}_1^2}{\hat{R}_0} + 2R_1\frac{\hat{R}_1}{\hat{R}_0} \quad (15)
\]

In the AR(1) case, reflection coefficients and normalized autocovariance functions are, except for the sign, exactly the same representation. Therefore, they feature the same characteristics and only the difference between averaging reflection coefficients and autocovariance functions will be analyzed.

Now, the expectations of the prediction error can be calculated for both methods of averaging. Suppose one wishes to average the information in two realizations denoted \(x_i\) and \(y_i\). The prediction error will be given by:

\[
PE_{RC} = R_0 + R_0\left(\frac{\hat{R}_{1x}}{\hat{R}_{0x}} + \frac{\hat{R}_{1y}}{\hat{R}_{0y}}\right)^2 - 2R_1\left(\frac{\hat{R}_{1x}}{\hat{R}_{0x}} + \frac{\hat{R}_{1y}}{\hat{R}_{0y}}\right) \quad (16)
\]

for averaging reflection coefficients and

\[
PE_{Cov} = R_0 + R_0\left(\frac{\hat{R}_{1x} + \hat{R}_{1y}}{\hat{R}_{0x} + \hat{R}_{0y}}\right)^2 - 2R_1\left(\frac{\hat{R}_{1x} + \hat{R}_{1y}}{\hat{R}_{0x} + \hat{R}_{0y}}\right) \quad (17)
\]

when covariances are averaged. Since these expressions are a function of stochastic variables, their expectation can be calculated using the Taylor series expansion, [11] pp. 53-54, are used, and the second order moments of the estimated autocovariance function, [11] pp. 324-328.

For an estimate of order \(1/m\), the expectation of the prediction error for an AR(1) model does not differ whether reflection coefficients or covariances are averaged:

\[
E\{PE_{RC}\} = E\{PE_{Cov}\} = R_0 + \frac{R_1^2}{mR_0} + \frac{R_1^2}{mR_0} \cdot \text{var}(\hat{R}_0) + \frac{1}{mR_0} \cdot \text{var}(\hat{R}_1) - 2\frac{R_1}{mR_0} \cdot \text{cov}(\hat{R}_0, \hat{R}_1) \quad (18)
\]

where \(n\) is the number of observations in one realization and \(m\) is the number of realizations. Although the prediction error estimates of order \(1/n\) are the same, the expectation of the estimated parameter, after averaging, shows a difference:

\[
E\{\hat{a}_{RC}\} = a \cdot \left[1 - \frac{2}{n}\right] \quad (19)
\]

\[
E\{\hat{a}_{Cov}\} = a \cdot \left[1 - \frac{2}{n \cdot m}\right]
\]

Formula (19) shows that the parameter bias is independent of the number of averaged realizations when averaging reflection coefficients, but it decreases when averaging the autocovariance functions of multiple realizations. The difference appears in the expectation of the prediction error only at order \(1/n^2\). Using Taylor series again, for averaging reflection coefficients can be found:

\[
E\{PE_{RC}\} = R_0 + \frac{R_1^2}{mR_0} + \frac{1}{mR_0} \cdot \text{var}(\hat{R}_0) + \frac{1}{mR_0} \cdot \text{var}(\hat{R}_1) - 2\frac{R_1}{mR_0} \cdot \text{cov}(\hat{R}_0, \hat{R}_1) \quad (20)
\]

and for averaging autocovariance functions:

\[
E\{PE_{Cov}\} = R_0 + \frac{R_1^2}{mR_0} + \frac{1}{mR_0} \cdot \text{var}(\hat{R}_0) + \frac{1}{mR_0} \cdot \text{var}(\hat{R}_1) - 2\frac{R_1}{mR_0} \cdot \text{cov}(\hat{R}_0, \hat{R}_1) \quad (21)
\]

in which the fourth order moments of \(\hat{R}\) are of order \(1/n^2\). Analysis of expressions (20) and (21) shows, that averaging reflection coefficients will lead to a higher prediction error than averaging autocovariance functions. In the averaged reflection coefficient case, the part of order \(1/n^2\), given by the fourth order moments of the autocovariance function, decays with \(1/m\), while the same part decays with \(1/m^2\) when averaging autocovariance functions. This means that the prediction error after averaging autocovariance functions will be close to the asymptotic \(1/n\) theory [6], while averaging reflection coefficients introduces a more significant term.

Using the fourth order moments of the estimated autocovariance function it is possible to find simpler expressions for the expectation of the prediction error. For averaging reflection coefficients is found for the AR(1) case:

\[
E\{PE_{RC}\} = R_0 \left(1 - a^2\right) \left[1 + \frac{1}{n \cdot m} + \frac{6}{n^2 \cdot m} \cdot \frac{1 + 5a^2}{(1 - a^2)^2} \left(1 + a^2\right)\right] \quad (22)
\]

and for averaging autocovariance functions:

\[
E\{PE_{Cov}\} = R_0 \left(1 - a^2\right) \left[1 + \frac{1}{n \cdot m} + \frac{6}{n^2 \cdot m} \cdot \frac{1 + 5a^2}{(1 - a^2)^2} \left(1 + a^2\right)\right] \quad (23)
\]

Using these expressions, a theoretical value for the expectation of the prediction error of order \(1/n^2\) can be estimated, when the generating process is known.

### 4.3 Simulation Results

Simulation experiments have been carried out to verify the theoretical \(1/n^2\) results of expressions and (22) and (23). In
these simulation experiments, multiple realizations of an AR(1) process have been generated for parameters in the range from -0.9 to 0.9. AR(1) models have been estimated using the Burg method. For different numbers of observations \( n \) and realizations \( m \), reflection coefficients and autocovariance functions have been averaged and the prediction error has been calculated by expression (2) over 10,000 simulation runs. Those values have been compared to the theoretically calculated values of the expectation of the prediction error, given in expressions (22) and (23). The results are displayed in Figure 1.

From Figure 1 can be seen that, in case of a parameter close to the unit circle, both theoretical and simulation results show that averaging autocovariance functions will lead to the model having the lowest prediction error and therefore the best spectral estimate.

Although theoretical analysis will be very complex, simulation experiments have shown that also for AR processes with order \( p > 1 \), averaging reflection coefficients leads to a higher prediction error than averaging autocovariance functions. When considering AR process with order \( p > 1 \), the theoretical complexity is due to an extra Taylor bias, which must be taken into account when averaging autocorrelation functions. This Taylor bias resembles the Taylor bias that has been taken into account when evaluating the average of reflection coefficients in the above described derivation.

5 Clutter rejection algorithm

An algorithm is presented that rejects clutter on the basis of its autocovariance structure. Since the clutter has a highly stochastic structure, parametric estimation methods are well suited.

The property of the clutter, used for clutter suppression, is that clutter is stationary over a considerable range. This means that the clutter in a range cell has about the same Doppler spectrum as the clutter in adjacent range cells.

The processing chain of the clutter rejection algorithm, proposed in this paper, is depicted in Figure 2.

The algorithm estimates parametric models in the range cell of interest, denoted by Cell\(_N\) in Figure 2, and in a number of range cells in its immediate environment. Because of the stationary properties of the clutter in range, the information in the adjacent range cells is averaged in order to obtain a clutter reference model.

The inverse of this clutter reference model is used to filter the clutter out of the cell of interest. Finally, detections are made in the Doppler spectrum from which the clutter is removed. All steps of this algorithm are discussed in more detail below.

5.1 Clutter reference model estimation

Since the Doppler structure of the clutter in the environmental range cells is representative for the clutter in the cell of interest, a clutter reference model can be estimated by averaging the autocovariance functions of these environmental cells.

In all adjacent cells, that have been chosen as an environment to Cell\(_N\), the reflection coefficients of AR models of order \( n-2 \) are estimated by Burg. For each cell these reflection coefficients are transformed into a covariance sequence. The mean of these covariance sequences is taken, resulting in an average covariance sequence, which is in turn transformed into reflection coefficients. Order selection is applied and the significant coefficients are transformed into a parameter vector of length \( p \), being the model order. These parameters represent the reference clutter model and they are the parameters of the clutter filter.

The choice of the size of the environment, used for the clutter reference model, is still a topic of research. Since a target is spread over some range cells, the directly adjacent cells should not be taken into the clutter model. This would cause the target to be suppressed too. Furthermore, enough range cells should be averaged to obtain a model that does
not suffer much from statistical fluctuations.

5.2 Parametric filtering

The clutter reference model is used as an inverse filter. This is established by replacing the poles of the clutter model by zeros, in order to compensate for the clutter peaks in the cell of interest.

In the z-domain, the output model of the filter can be written as a convolution of the input model and the filter itself. Using the z-notations for the AR model of the cell of interest, \( \hat{A}_{cell}(z) \), and for the inverted clutter reference model, \( \bar{A}_{clutter}(z) \), a filtered model results as follows:

\[
\text{filtered model} = \frac{\bar{A}_{clutter}(z)}{\hat{A}_{cell}(z)}.
\]

In order to get the most detailed filtered model, the cell model of order \( n - 2 \) is filtered by the clutter model of order \( p \). This leads to an ARMA model of order \((n-2, p)\), to which order selection has to be applied.

However, an ARMA model cannot simply be reduced to lower order models. Therefore, the autocovariance function of the model is calculated and an AR model, to which order selection can be applied, is inferred from this autocovariance function. First, the ARMA model, given by:

\[
x_t + a_1 x_{t-1} + ... + a_p x_{t-p} = \varepsilon_t + b_1 \varepsilon_{t-1} + ... + b_q \varepsilon_{t-q},
\]

is split in an MA part and an AR part by the use of intermediate series \( \varepsilon_t \), as depicted in Figure 3.

![Figure 3 Split ARMA model for calculation of the autocovariance function of \( x_t \).](image)

The AR part of the ARMA model is given by:

\[
v_t + a_1 v_{t-1} + ... + a_p v_{t-p} = \varepsilon_t,
\]

and the MA part is given by:

\[
x_t = v_t + b_1 v_{t-1} + ... + b_q v_{t-q}.
\]

The autocovariance function \( R_{vv}(k) \) of the intermediate series \( v_t \) can be calculated from the Yule-Walker equations:

\[
R_{vv}(k) = \begin{cases} 
\sigma_e^2 \cdot \prod_{i=1}^{p} \frac{1}{1-|k|} & k = 0 \\
- \sum_{i=1}^{k} \hat{a}_k, i \hat{R}_{xx}(k-l) & k \geq 1
\end{cases},
\]

where reflection coefficients \( k_i \) are calculated from parameters \( a_i \) by inverting expression (8).

Since \( x_t \) is a linear combination of \( v_t \), the autocovariance function \( R_{xx}(k) \) of \( x_t \) is calculated by pre-multiplying expression (27) by \( x^*_{t-k} \) and rearranging it to:

\[
R_{xx}(k) = \begin{bmatrix} 1 & b_1^* & ... & b_q^* \end{bmatrix}.
\]

\[
\begin{bmatrix}
R_{vv}(k) & R_{vv}(k-1) & ... & R_{vv}(k-q) \\
R_{vv}(k+1) & R_{vv}(k) & ... & 0 \\
& & \vdots & \ddots & \vdots \\
R_{vv}(k+q) & & ... & R_{vv}(k) & b_q
\end{bmatrix} \begin{bmatrix} 1 \ b_1^* \ ... \ b_q^* \end{bmatrix}.
\]

The AR model, to which order selection can be applied, is calculated from this autocovariance function \( R_{xx}(k) \) by the Levinson-Durbin recursion (9).

Without further pre-processing the algorithm is now applied to the data removing the clutter reflections. The experiments with real-life data in section 6 will show that only the reflections, identified as being due to possible targets are left. The reflections due to the clutter are effectively eliminated by the clutter filter algorithm.

5.3 Detection

Since the final autoregressive model provides a rational description of the spectrum in expression (24), it is possible to find peak positions analytically. Therefore, the derivative of the spectrum with respect to \( \omega \) has to be set to zero. The power spectrum of an AR model is generally defined as:

\[
h(\omega) = \frac{\sigma_e^2}{2\pi} \cdot \frac{1}{1 + a_1 e^{-j\omega} + \ldots + a_p e^{-j\omega}}
\]

This provides a parametric rational description of the spectrum. Therefore it is possible to find the peaks in the spectrum analytically, using the first derivative with respect to \( \omega \). The AR(1) situation will be used as an example that can be generalized to AR models of arbitrary order \( p \). For an AR(1) model, the spectrum (30) can be written as:

\[
h(\omega) = \frac{\sigma_e^2}{2\pi} \cdot \frac{1 + \sum_{i=1}^{p} a_i e^{-j\omega}}{1 + \sum_{i=1}^{p} a_i^* e^{j\omega}}
\]

Therefore the first derivative with respect to \( \omega \) of the spectrum is given by:

\[
\frac{\partial}{\partial \omega} h(\omega) = \frac{\sum_{i=1}^{p} j a_i e^{-j\omega}}{(1 + \sum_{i=1}^{p} a_i^* e^{j\omega})^2} \cdot \frac{p \sum_{i=1}^{p} a_i e^{-j\omega}}{(1 + \sum_{i=1}^{p} a_i^* e^{j\omega})^2}
\]

The roots of this expression are the locations of the extremes. A simple check results in the peaks of the spectrum.
However, not all spectral peaks represent targets. Many peaks are due to noise or to the remains of the clutter. Therefore, a technique is needed to indicate whether peaks are relevant enough to cause a detection. The relevance is determined by the desired probability of false alarm given a probability of detection. Many algorithms exist, to indicate if peaks are relevant enough to cause a detection. For instance a CFAR algorithm can be used. Another possibility is an algorithm that only uses the Doppler spectrum of one range cell. Then, quantities like the power of a peak above the noise background or above the average signal power, can be used. Another possibility is the sharpness of a peak, indicated by the second derivative. Application of the second derivative algorithm to the filtered range-Doppler spectrum of Figure 6 and Figure 8 leads to the detections which are indicated by crosses.

6 Analysis of registered radar data

The algorithm has been tested on numerous loggings of an X-band radar, yielding 21 pulses per burst. It will be shown that the performance of the algorithm is independent of the situation. It performs as well on sea as on land clutter caused by vegetation. The only restriction is the stationarity of the clutter in range.

6.1 Sea clutter

Sea clutter has a highly stochastic Doppler structure, caused by the stochastic motions of the sea waves. This results in a wide Doppler spectrum. The Doppler peak frequency is in general not zero, depending on the direction in which the waves move, relative to the radar. In these examples, 21 sweeps of a radar that was situated at land, looking at the sea, were used. In this situation, the waves were incoming, which leads to positive Doppler frequencies.

![Figure 4 Autoregressive range-Doppler spectrum.](image)

In Figure 4, the autoregressive spectral estimate of the data is depicted. In this figure, the horizontal axis denotes the Doppler speed in m/s and the vertical axis indicates the range. The bar indicates power in dB. This figure shows that the AR estimate leads to a high resolution in the range-Doppler spectrum. Target and clutter positions are well defined and the background noise is low.

Figure 5 depicts the environmental clutter reference model, which results after averaging the information in a number of adjacent range cells. The figure shows peaks in the clutter model where they are in the original range-Doppler spectrum. Furthermore, it can be seen that there is a ‘shadow’ of the target. However, this ‘shadow’ appears at a different range in the clutter model than it is in the original spectrum. Therefore, it will not lead to suppression of the target.

Figure 6 shows the filtered range-Doppler spectrum. In this spectrum, the clutter is suppressed effectively, while the targets are not suppressed. It is obvious that it is easy to make detections in this filtered range-Doppler spectrum. Based on this spectrum, detections have been made. First, by setting the derivative of the power spectral density with respect to \( \omega \) at zero, the positions of all peaks are determined analytically. Then, the second derivative is used as a measure for the sharpness of peaks. A threshold is used to discriminate detections from peaks due to clutter remains and noise. The result are the crosses.

![Figure 6 Filtered range-Doppler spectrum and detections, indicated by crosses.](image)

The figure shows that both targets are detected, while there were no false alarms on the sea clutter.

6.2 Land clutter

Without modification, the algorithm is applied to X-band
radar loggings of 1 person in the woods. Now, 63 pulses per burst have been used.

Land clutter has a narrow Doppler spectrum, see Figure 7, which is always centered around 0 m/s. Therefore, targets with a Doppler speed close to 0 m/s will be hard to detect. When comparing this spectrum to the spectrum in Figure 4, the narrow band nature of the land clutter Doppler spectrum can be seen.

It can be seen that the widening of the spectrum at range 1.3 km, due to a walking person, is visible at neighboring range cells. Applying the clutter rejection filter results in the range-Doppler spectrum of Figure 8. In this figure, the land clutter is suppressed effectively, leaving the reflection of the person.

The same detection algorithm is applied to this data, as shown in Figure 7, showing that also land clutter is rejected effectively, while targets are detected.

7. Conclusions

An algorithm is presented that applies high resolution spectral estimation methods to radar Doppler processing. Besides a high resolution spectral estimate, a reference model of the clutter is estimated. By means of a parametric filter, the clutter is rejected effectively, while leaving target reflections unaffected. The parametric description of the filtered Doppler spectrum allows for very accurate detections, while the clutter rejection algorithm provides less false alarms on clutter.

8 References