A Suboptimum Permutation Test for Radar Detection in Log-Normal Clutter Environments

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Abstract.

In this paper we have used a coherent log-normal model for the radar clutter: the in-phase and quadrature components of clutter have been modeled to give a log-normal amplitude distribution and a Gaussian distribution of the phase. We have compared this model with another one in which the distribution of the phase is uniformly distributed.

Also, we present detectability curves of the permutation test under log-normal noise environments and different types of target models (nonfluctuating, Swerling I and Swerling II). We analyze the detector performance in terms of detection probability ($P_d$) versus signal-to-noise ratio (SNR) for different parameter values: the integrated pulse number $N$, the noise reference ratio (SNR) for different parameter values: the probability, clutter model, log-normal clutter.

Keywords: nonparametric tests, permutation test, radar detection, false alarms, detection probability, clutter model, log-normal clutter.

1. -Introduction and Preliminaries.

Radar detection uses concepts of information fusion, because the underlying signals are considered in time/space and frequency domains. Radar deals with signal, noise and clutter from different reference cells (samples vectors) and, also, different doppler frequencies.

1.1.- Permutation Test

The permutation test is a binary nonparametric test, which is distribution-free under independent and identically distributed (IID) samples [1,2,3].

The distribution of a block of IID samples is invariant under the permutation of its sample components. That is, consider a sample vector $(x_1,x_2,...,x_n)$ of $n$ IID samples where $F_0(x)$ is the distribution function of a sample, if

$$F(x_1,x_2,...,x_n)= F_0(x_1)F_0(x_2)...F_0(x_n),$$

then

$$F(x_1,x_2,...,x_n)=F(x_2,x_1,...,x_n)=..(n!),=F(x_{(1)},x_{(2)},x_{(3)})$$

To generate a permutation test, the sample space $R^n$ is partitioned into $n!$ regions $D_i$ ($i=1,2,...,n!$) where

$$D_i = \{ x=(x_1, ..., x_n): if \ x \in D_i, \ x \ (permutation) \not\in D_i \}$$

in such a way that

$$D_i \cap D_j = \emptyset \ i,j=1,2,..,n! \ i \neq j \ and \ \bigcup_{i=1}^{n!} D_i = R^n$$

Each sample vector $x$ belongs to one of these regions $D_i$ ($i=1,2,...,n!$), and we can get a different vector by permuting their components, each one belonging to one different region $D_i$. It is possible to partition $R^n$-space in different ways in order to fulfill $D_i$-conditions.

Under the null hypothesis $H_0$ (target absent), the probability that the sample vector $x_i$ belongs to one of the regions $D_i$ is $1/n!$, i.e. $\text{Prob}\{x \in D_i\} = 1/n!$.

Under the alternative hypothesis $H_1$ (target present), there are $D_i$-regions with more probability measure than other ones and now the probability that $x \in D_i$ ($i=1,...,n!$) is not uniform.

Given a $D_i$-partition, we define the decision region as the union of $K$ regions $D_i$. In order to get the maximum probability of detection we select the $D_i$-regions with largest probabilities. Just under $H_{1n}$, the false alarm probability $P_{fa}$ is $K/n!$, where $K$ is the number of $D_i$-regions. The optimum
permutation test would be the partition that achieves a maximum detection probability.

In radar applications, we have $N$ sample vectors $x_i$, $x_2$, ..., $x_N$ where $N$ is the number of pulses per antenna beamwidth. Each sample vector $x_i$ has $M$ noise reference samples $x_{ij}$, $j=1,2,...,M$ and the sample under test $x_i$, i.e. $x_i = (x_{i1}, x_{i2}, ..., x_{iM}, x_{i0})$. Under the null hypothesis $H_0$ (target absent) we suppose that the components of $x_i$ are IID, but under the alternative hypothesis $H_1$ (target present) they are not IID (reference samples $x_{ij}$ are IID and $x_i$ has different distribution of $x_{i0}$, $j=1,2,...,M$).

Now, the distributions associated with $H_0$ is

$$f(x_i/H_0) = \sum_{i=0}^{N} \prod_{j=1}^{M} f_{ij}(x_{ij}) \cdot f_{i0}(x_{i0}) \quad (1a)$$

where $f_{ij}(\cdot)$ is the probability density function of a noise sample in the $i$th-pulse. Under $H_1$, we have

$$f(x_i/H_1) = \sum_{i=0}^{N} \prod_{j=1}^{M} f_{ij}(x_{ij}) \cdot f_{i0}(x_{i0}) \quad (1b)$$

where $f_{ij}(x_{ij})$ is the probability density function of a sample under test $x_i$ (signal + noise) in the $i$th-pulse.

In order to test $H_0$ against $H_1$ in Neyman-Pearson sense, we take the likelihood ratio

$$\frac{f(x_i/H_1)}{f(x_i/H_0)} = \sum_{i=0}^{N} \prod_{j=1}^{M} f_{ij}(x_{ij}) \cdot f_{i0}(x_{i0}) \quad (2a)$$

In the case of Gaussian noise conditions and nonfluctuating target models, applying the last expression at the output of the envelope detector, we have (after taking Neperian logarithm):

$$\sum_{i=1}^{N} \ln \frac{f(x_i/H_1)}{f(x_i/H_0)} = \sum_{i=1}^{N} \ln \frac{f_{ij}(x_{ij})}{f_{i0}(x_{i0})} \quad (1d)$$

We optimize the permutation test by permuting all the samples in each vector $x_i=(x_{i1}, x_{i2}, ..., x_{iM}, x_{i0})$, $i=1,2,...,N$ and selecting the upper results. The number of $K$ higher results selected depends on the false alarm probability $P_{fa}$, i.e. $P_{fa} = K/(M+1)^K$, where $K$ is the number of $D_i$-regions associated with upper results after doing permutations. In [4] we have presented a Permutation Test algorithm for Radar Detection; in [6] we have compared Permutation Test against Rank Test applied to nonparametric radar detection under Gaussian noise environments.

### 1.2.- Log-Normal Clutter

In several applications of radar detection, the clutter amplitude is not distributed as Rayleigh: the amplitude distributions have long tails specially when we are working with high resolution radar, low grazing angles, and horizontal polarization at high frequencies.

In [7] we analyze a detector based on the optimum permutation test in the Neyman-Pearson sense, under Weibull noise environments. Now, we analyze the permutation test in log-normal clutter environments which are present in high resolution radar systems. The permutation test is applied to the amplitude signal (after the envelope detector).

The clutter has been modeled as a coherent process (i.e. both the amplitude and the phase was considered) and the thermal noise has been neglected. We have considered linear, quadratic and logarithmic statistics for the permutation test, and they have been compared in terms of detection performance. We analyze the detector performance of permutation tests in terms of detection probability ($P_d$) versus signal-to-clutter (noise) ratio ($SNR$), considering the number of integrated pulses ($N$), the number of noise reference samples ($M$) and the false alarm probability ($P_{fa}$) as parameters.

We say that $R$ is a log-normal random variable if its logarithm is distributed as a Gaussian (normal)

$$f(R) = \frac{1}{\sqrt{2\pi} \sigma R} \exp \left(-\frac{1}{2\sigma^2} \ln^2 \left(\frac{R}{R_m}\right)\right) \quad (2a)$$

random variable. If $r = \ln(R)$ is a normal random variable with mean $\mu$ and standard deviation $\sigma$, then $R$ is a log-normal distribution with probability density function (pdf), given by

$$\ln(r) = \mu$$

A log-normal coherent variable $w$ is a complex variable [8]: $w = u + jv$, where $u$ and $v$ are the real and imaginary components related as follows

$$w = u + jv = e^{exp(z)} = \exp(x) \cdot \{\cos(y) + j\sin(y)\} \quad (2b)$$
where $x$ and $y$ are independent Gaussian variables, $z = x + jy$ (complex Gaussian variable), $\exp(\cdot)$ is the complex exponential. So then
\[
\begin{align*}
u &= \exp(x) \cos(y) \\
\arg(w) &= y
\end{align*}
\] (3a) and
\[
|w| = \exp(x) \\
\text{arg}(w) = y
\] (3b)

The envelope of $w$, $|w|$, is log-normal while the phase of $w$, arg$(w)$, is Gaussian.

On the other hand, the pdf of $u$ and $v$ is given in [8]. The phase of the $w$ is not uniformly distributed, then the log-normal model is not a Spherically Invariant Random Process (SIRP), and has not got a circular symmetry in the random variables. We have changed the distribution of the clutter phase by another one uniformly distributed in order to compare both models, and show the differences between them. Note that the model with phase of non-uniform distribution is not representative of real models. We will show that results are different for the two models (Gaussian phase and uniform phase) depending on the median value of the log-normal distribution and, also, the target model.

Note that the log-normal distribution depends on two parameters: the median and the standard deviation, both are relevant for the clutter power. We will consider the standard deviation equals to 1 and increase median value with clutter power; or, on the contrary, we will fix median and increase standard deviation with clutter power.

2. -Generation of Log-Normal Clutter.

The model for generating a log-normal random variable (rv) starts from the generation of uncorrelated Gaussian rv’s that are applied to a non-linear system. This model can be extended to the generation of correlated sequences. The model that we are used [8] is a linear filter cascaded with a nonlinear transformation like the complex exponential function. We show this model in Figure 1, where Q and I are independent white Gaussian sequences.

![Diagram](Figure 1: Structure for obtaining (U,V) from (I,Q) sequences)

The correlation introduced by the filter is related to the covariance matrix, as follows
\[
M_c = \begin{pmatrix} R_c(0) & R_c(-1) & \cdots & R_c(-p) \\
R_c(0) & R_c(0) & \cdots & R_c(-p) \\
\vdots & \vdots & \ddots & \vdots \\
R_c(p) & R_c(p) & \cdots & R_c(0) \\
\end{pmatrix}
\]

where $R_c(k)$ is the clutter autocorrelation function.

We have used the autocorrelation function, defined as follows
\[
R_c(k) = R_c(0) \cdot \rho_c^{|k|} \cdot e^{-|2\pi f_c| \cdot T} \quad (4)
\]

where $R_c(0)$ is the clutter power,
\[
\rho_c \text{ is the correlation coefficient between two consecutive samples,}
\]
\[
f_c \text{ is the mean clutter Doppler frequency,}
\]
\[
T \text{ is the time interval between two consecutive pulses.}
\]

The linear filter of Figure 1 is fed by white Gaussian noise $n_i$ (I or Q). So the output $s_i$ (X or Y) is given by
\[
s_i = n_i - \sum_{k=1}^{p} a_{pk} \cdot s_{i-k}
\]

The design of the Autoregressive model will be done by the Yule-Walker equations.

\[
M_c \cdot \begin{pmatrix} a_{p1} \\
\vdots \\
a_{pp} \end{pmatrix} = \begin{pmatrix} \sigma^2 \\
\vdots \\
0 \end{pmatrix} \quad (5)
\]

where $\sigma^2$ is the noise variance of the input (white Gaussian input sequence).
To solve the equations (5) we use the Levinson-Durbin algorithm, presented in Figure 2 (see last page).

3.-Computer Simulation Results.

As we know $x$ and $y$ are two Gaussian rv’s with zero mean, and we have defined the complex log-normal rv as $w=u+iv=\exp(x+jy)$. The amplitude of $w$ depends only of $x$, and is log-normal; the phase of $w$ is equal to the Gaussian $y$. Now, we consider the detection of targets in log-normal noise. The detection probability ($P_d$) versus signal-to-noise ratio (SNR) has been obtained by simulations for the permutation test, considering as parameter false alarm probability ($P_{fa}$), number of integrated pulses ($N$) and number of noise reference samples ($M$).

First, we compare a noise model that has a uniformly distributed phase in $[0,2\pi]$ and a noise model of Gaussian phase (coherent log-normal).

In Figures 3-5, we show results of $P_d$ versus SNR under log-normal noise environments for the logarithmic permutation detector (the test statistic of the permutation test is logarithmic). Also we suppose uncorrelated noise ($\rho=0$).

As it can be seen from Figures 3, we get differences of 1 dB between both types of phases for the nonfluctuating target. On the other hand, from Figures 4 and 5, there are no differences with other target types.

Also, it is interesting to show $P_d$ versus SNR with the median value as a parameter. In Figures 6 and 7 we show the results for both models of the phase (Gaussian and uniform) for the log-normal noise and nonfluctuating target model.

Figure 3. Detection probability ($P_d$) versus signal-to-noise ratio (SNR). Parameters: $N=M=8$, $P_{fa}=10^{-6}$ and $10^{-7}$, nonfluctuating target model. Logarithmic permutation detector. Median=1 (Log-normal noise with $\rho=0$).

Figure 4. Detection probability ($P_d$) versus signal-to-noise ratio (SNR). Parameters: $N=M=8, P_{fa}=10^{-6}$ and $10^{-7}$, Swerling I target model. Logarithmic permutation detector. Median=1 (Log-normal noise with $\rho=0$).

Figure 5. Detection probability ($P_d$) versus signal-to-noise ratio (SNR). Parameters: $N=M=8, P_{fa}=10^{-6}$ and $10^{-7}$, Swerling II target model. Logarithmic permutation detector. Median=1 (Log-normal noise with $\rho=0$).

Figure 6. Detection probability ($P_d$) versus signal-to-noise ratio (SNR). Parameters: $N=M=8, P_{fa}=10^{-7}$, nonfluctuating target and Gaussian phase. The
median value as a parameter. Logarithmic permutation detector. (Log-normal noise with $\rho=0$).

**Figure 7.** Detection probability ($P_d$) versus signal-to-noise ratio (SNR). Parameters: $N=M=8$, $P_f=10^{-7}$, nonfluctuating target and uniform phase. The median value as a parameter. Logarithmic permutation detector. (Log-normal noise with $\rho=0$).

As it can be seen from Figures 6 and 7, the detection probability curves are more sensitive with respect to the median value for the Gaussian phase than for the uniform phase, and when the median value is 0.8 there is no difference (curves $P_d$ versus SNR are similar).

Now, we analyze the influence of linear, quadratic and logarithmic statistics for the permutation test, and compare each other in terms of detection probability. Figure 8 shows the detectability for nonfluctuating targets with $N=M=8$, $P_f=10^{-6}$ and $10^{-7}$, the median equals 0.8, and uncorrelated clutter ($\rho=0$).

**Figure 8.** Detection probability ($P_d$) versus signal-to-noise ratio (SNR) for nonfluctuating target model. Parameters: $N=M=8$, $P_f=10^{-6}$. Different statistics (linear, quadratic and logarithmic) for the permutation test. (log-normal noise with $\rho=0$).

Now, curves of $P_d$ versus SNR is given in the Figures 9-11, taking the clutter correlation coefficient (?) as a parameter. Note that $P_d$ increases as ? increases but also $P_f$ increase because the permutation test is not distribution-free [1-3] for $\rho$.?

**Figure 9.** Detection probability ($P_d$) versus signal-to-noise ratio (SNR) for nonfluctuating target. Parameters: $N=M=9$, $P_f=10^{-7}$ ($\rho=0$). Clutter correlation coefficient $\rho$; $0=\rho=1$.

**Figure 10.** Detection probability ($P_d$) versus signal-to-noise ratio (SNR) for Swerling I target. Parameters: $N=M=9$, $P_f=10^{-7}$ ($\rho=0$). Clutter correlation coefficient $\rho$; $0=\rho=1$.

Finally, we show the influence of linear, quadratic and logarithmic statistics for the permutation test under correlated clutter. Figure 12 show the detectability for nonfluctuating targets with $N=M=8$, $P_f=10^{-6}$ and median equals 0.8, taking the clutter correlation coefficient (?) as a parameter.
4. - Conclusion

In this paper, some results of detection probability ($P_d$) versus signal-to-noise ratio (SNR) have been presented for the permutation test under log-normal clutter (noise) environments and different detector parameters.

The clutter models with uniform and Gaussian phase distributions are equivalent if the median is equal to 0.8.

Also, it is apparent that the logarithmic permutation detector (the test statistic is logarithmic) performs better than the linear or quadratic permutation detectors under log-normal correlated clutter.

Finally, the permutation test is nonparametric (distribution-free procedure) for the uncorrelated noise (clutter). For correlated clutter the permutation test should be modified in order to maintain its nonparametric character (although from recent results, the permutation test is robust for moderate correlation coefficient $0 \leq \rho \leq 0.8$).

References.


Figure 2: Algorithm of Levison-Durbin for the calculation of $a_n$, $n=1,2,...,p$ coefficients of (order-p) filter.