Accurate Murty’s Algorithm for Multitarget Top Hypothesis Extraction

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Abstract—In most hypothesis-oriented Multiple Hypothesis Tracking (MHT) implementations, the target-to-measurement data association is typically solved by using the Murty’s algorithm. However, the Murty’s algorithm has no control over the diversity of target-to-measurement associations - often the top associations vary only slightly. In addition, in practical tracking solutions, tracks are often grouped as tentative or continued. It was observed with real data sets that in the associations, the top hypotheses consist of mostly similar associations with the same confirmed tracks along with some permutations of new measurements. The result is that a fixed set of confirmed tracks dominate diversity of the association tree. To overcome this problem, a modified Murty’s algorithm, which can achieve any user defined (or adaptable) diversity of track-to-measurement association of different types of tracks, is proposed in this paper. Numerical examples are provided to demonstrate the improved efficiency in hypotheses generation by the proposed method.

Keywords: K-best assignment, Murty’s algorithm, data association.

I. INTRODUCTION

One of the most important stages in multiple target tracking problems is target-to-measurement data association. The hypothesis-oriented Multiple Hypothesis Tracking (MHT) tracker was proposed in [15] to resolve the target-to-measurement data association in multiple target tracking problems in cluttered environment. The original MHT is theoretically the best approach for multiple target tracking problems. However, the exponential growth in the number of hypotheses has raised doubts about its practical feasibility.

In [12], an algorithm that efficiently generates the K-best solutions in assignment problems, called the Murty’s algorithm, was proposed. Murty’s algorithm has been widely used to generate the K-best assignments in multitarget tracking [4, 5, 8, 10], making the MHT feasible in practice. In [6, 7], Reid’s multiple hypothesis tracking algorithm was reformulated by exploiting a K-best ranked linear assignment algorithm for data association based on Murty’s algorithm. In [13, 14], three optimized implementations of Murty’s algorithm in which the input cost matrix for Murty’s algorithm is formed by appending dummy values to the standard target-to-measurement cost matrix are discussed. The major drawback of the standard Murty’s algorithm is that it has no control over the diversity of target-to-measurement associations — often the top associations vary only slightly. In practical tracking solutions, tracks are often grouped as tentative (being initialized) or continued (being maintained). It was observed that in the associations observed with real data sets, the top hypotheses consist of mostly similar associations with the same confirmed tracks along with some permutations of new measurements. The reason is that, in the standard Murty’s algorithm, all tracks are equally used to partition the feasible solution space of feasible data associations. In addition, trackers usually generate lots of tentative tracks, most of which are in fact false tracks, in high clutter areas. In such situations, the confirmed tracks will lose the competition for data association diversity with the tentative tracks by using the standard Murty’s algorithm. That is, the data associations vary only slightly for the confirmed tracks in the top K assignments, which in turn degrades the performance of the tracker. To overcome this drawback of the standard Murty’s algorithm, a modified Murty’s algorithm, in which various priorities are assigned to different types of tracks so that the data association diversities of different types of tracks can be controlled, is proposed. The underlying idea of the modified algorithm is to use only a certain type of tracks, instead of all the tracks, to partition the solution space. For example, higher priority can be assigned to confirmed tracks in high clutter area and only confirmed tracks are used in the partition procedure so that the data association diversity of confirmed tracks can be ensured in the top K assignments. It is also worth noting that the standard Murty’s algorithm is a special case of the modified Murty’s algorithm by assigning equal priority to all the tracks. This paper aims at improving the performance of the hypothesis-oriented MHT through using the modified Murty’s algorithm. The comparison between the track-oriented MHT and the hypothesis-oriented MHT using the proposed algorithm is beyond the scope of this paper, and the exploration of merits of the proposed algorithm w.r.t the track-oriented MHT remains as a possible future work.

This paper is organized as follows. Section II reviews the standard Murty’s algorithm and its application in assignment generation in MHT tracker. Section III presents the proposed modified Murty’s algorithm. The application of the modified Murty’s algorithm is demonstrated in Section IV. Simulation results are presented in Section V.
II. THE STANDARD MURTY’S ALGORITHM FOR MHT TRACKERS

A. Review of Murty’s Algorithm

In this subsection, a short summary of the standard Murty’s algorithm is provided. The reader is referred to [12] for details on the standard Murty’s algorithm. The implementation of the Murty’s algorithm has the following iterative steps:

1) **Stage 1:** Find the best assignment $a_1^*$ using an optimal assignment algorithm such as Auction algorithm [2] or Jonker-Volgenant-Castanon (JVC) algorithm [9, 11].

$$ a_1^* = \{(i_1,j_1), \ldots, (i_n,j_n)\} $$

Denote by $N_1, N_2, \ldots, N_{n-1}$ the nodes in stage 1.

$$ N_1 = \{(i_1,j_1)\} $$

$$ N_2 = \{(i_1,j_1),(i_2,j_2)\} $$

$$ \vdots $$

$$ N_{n-1} = \{(i_1,j_1),(i_2,j_2),\ldots, (i_{n-1},j_{n-1})\} $$

Then $\{N_1, \ldots, N_{n-1}\}$ forms a list of nodes for stage 1. Denote by $\mathcal{A}$ the collection of all the assignments where $e$ denotes the number of current nodes. Each node $N_i$ is a non-empty subset of $\mathcal{A}$. All nodes are mutually disjoint and their union is $\mathcal{A}/\{a_1^*\}$.

2) **General stage $k$:** Suppose $a_1^*, \ldots, a_{k-1}^*$ and $N_1, \ldots, N_k$ have been determined for this stage. Denote by $a_k$ the best assignment in node $N_k$ and $Z(a_k)$ the corresponding cost, then the global $(k+1)$-th best assignment $a_{k+1}^*$ is given as

$$ i^* = \arg \min_i Z(a_i) $$

$$ a_{k+1}^* = a_i^* $$

Denote by $\{N_{i_k^*}, \ldots, N_{i_k^*}\}$ the set of nodes resulting from the partition of $N_k$ by $a_{k+1}^*$. Then the nodes for next stage are $\{N_1, \ldots, N_{k-1}, N_{i_k^*+1}, \ldots, N_{i_k^*+1}, \ldots, N_{i_k^*}\}$.

B. Assignment Generation in MHT with the Standard Murty’s Algorithm

As indicated in [1, 13], the general form of the cost matrix in the $K$-best MHT is given as

$$ C = \begin{pmatrix}
    l_{i_1,1} & \cdots & l_{i_1,n} & \lambda_N & \infty & \infty & \lambda_F & \infty & \infty \\
    \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
    l_{i_m,1} & \cdots & l_{i_m,n} & \infty & \lambda_N & \infty & \cdots & \lambda_F & \infty \\
    q & \infty & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\
    \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
    \infty & q & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\
    \infty & \infty & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\
    \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
    \infty & \infty & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\
\end{pmatrix}
$$

where $m$ is the number of measurements and $n$ is the number of tracks. In the above, $l_{i,j}$ depends on the likelihood of associating the $i$-th measurement to the $j$-th track. Also, $\lambda_N$ and $\lambda_F$ depend on the new target density and clutter density, respectively, and $q$ represents the probability that no measurement is associated with current track. The bottom $m + n$ rows of the cost matrix correspond to the dummy measurements. The detailed equations for computing the cost matrix are given in [7, 13]. The assignments generated by the standard Murty’s algorithm suffer from low diversity in that the track-to-measurement associations of existing tracks might be identical and the only difference being in the association between new target and clutter. An example will illustrate this inefficient assignment generation problem. In this example, assume that there are two confirmed tracks $T_1$ and $T_2$, and four measurements $M_1 \sim M_4$. The corresponding cost matrix is assumed to be as follows:

$$ C_1 = \begin{pmatrix}
    0.2 & 0.8 & 4 & \infty & \infty & 3 & \infty & \infty \\
    0.86 & 0.23 & 4 & \infty & \infty & 3 & \infty & \infty \\
    4.1 & 4.9 & \infty & \infty & 3 & \infty & \infty & \infty \\
    4.7 & 5.1 & \infty & \infty & \infty & 4 & \infty & \infty \\
    10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 \\
    10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 \\
    \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
\end{pmatrix} $$

The 10-best assignments generated by the Murty’s algorithm are

$$ a_1^* = \{(1,1), (2,2), (3,9), (4,10), (5,3), (6,4), (7,5), (8,6) \}, \{(9,7), (10,8)\} $$

$$ a_2^* = \{(1,1), (2,2), (3,5), (4,10), (5,3), (6,4), (7,6), (8,7) \}, \{(9,8), (10,9)\} $$

$$ a_3^* = \{(1,1), (2,2), (3,9), (4,6), (5,3), (6,4), (7,5), (8,7) \}, \{(9,8), (10,10)\} $$

$$ a_4^* = \{(1,2), (2,1), (3,9), (4,10), (5,3), (6,4), (7,5), (8,6) \}, \{(9,7), (10,8)\} $$

$$ a_5^* = \{(1,1), (2,2), (3,5), (4,6), (5,3), (6,4), (7,7), (8,8) \}, \{(9,9), (10,10)\} $$

$$ a_6^* = \{(1,2), (2,1), (3,5), (4,10), (5,3), (6,4), (7,6), (8,7) \}, \{(9,8), (10,9)\} $$

$$ a_7^* = \{(1,2), (2,1), (3,5), (4,6), (5,3), (6,4), (7,7), (8,8) \}, \{(9,9), (10,10)\} $$

$$ a_8^* = \{(1,7), (2,2), (3,1), (4,10), (5,3), (6,4), (7,5), (8,6) \}, \{(9,8), (10,9)\} $$

$$ a_9^* = \{(1,2), (2,8), (3,1), (4,10), (5,3), (6,4), (7,5), (8,6) \}, \{(9,8), (10,10)\} $$

$$ a_{10}^* = \{(1,7), (2,2), (3,9), (4,1), (5,3), (6,4), (7,5), (8,6) \}, \{(9,8), (10,10)\} $$

and the corresponding costs are 6.43, 7.43, 7.43, 7.66, 8.43, 8.66, 9.66, 10.33, 10.9 and 10.93. The corresponding track-to-measurement associations are presented in Table I. As shown in Table I, the data association for the tracks is exactly the same in the top three best assignments, i.e., the data association diversity for the existing tracks is one within the top 3 best assignments. This low diversity implies an inefficient hypotheses generation for the existing tracks. As the number of measurements increases, this problem will become more severe (see Appendix). It can be observed from this example that the inefficiency is due to the two possible choices for every unassociated measurement, i.e., new track or clutter. One possible solution to overcome the low diversity problem is to initialize every unassociated measurement as a new track and apply some track management technique to delete the
The problem of inefficient assignment generation is resolved. However, a large number of tentative tracks will appear in the coming scan because all the unassociated measurements are initialized as new tracks, which are tentative. These tentative tracks will also compete for data association diversity with other existing tracks. For example, assume that there are two confirmed tracks $T^1$ and $T^2$, and five measurements $M_1, M_2, M_3, M_4$ and $M_5$. The corresponding cost matrix is constructed as follows:

$$C^* = \begin{pmatrix}
T^1 & T^2 & T^3 & T^4 & T^5 & T^{\text{new}} & T^{\text{new}} & T^{\text{new}} & T^{\text{new}} & T^{\text{new}} \\
M_1 & 0.30 & 4.70 & 4.92 & 4.91 & 5 & \infty & \infty & \infty & \infty \\
M_2 & 4.90 & 0.20 & 4.91 & 4.99 & \infty & 5 & \infty & \infty & \infty \\
M_3 & 4.10 & 4.90 & 4.89 & 4.93 & \infty & \infty & 5 & \infty & \infty \\
M_4 & 4.01 & 4.89 & 4.89 & 4.96 & \infty & \infty & \infty & 5 & \infty \\
M_5 & 4.20 & 4.79 & 4.87 & 4.97 & \infty & \infty & \infty & \infty & 5 \\
T^6 & 10 & \infty & \infty & \infty & 0 & 0 & 0 & 0 & 0 \\
T^7 & \infty & 10 & \infty & \infty & 0 & 0 & 0 & 0 & 0 \\
T^8 & \infty & \infty & 10 & \infty & 0 & 0 & 0 & 0 & 0 \\
T^9 & \infty & \infty & \infty & 10 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}$$

The top 5 assignments generated by using standard Murty’s algorithm based on the above cost matrix are:

$$a^*_1 = \{(1, 1), (2, 2), (3, 4), (4, 8), (5, 5), (6, 3), (7, 6), (8, 7), (9, 9)\}$$

$$a^*_2 = \{(1, 1), (2, 2), (3, 7), (4, 5), (5, 6), (7, 6), (8, 9), (9, 8)\}$$

$$a^*_3 = \{(1, 1), (2, 2), (3, 4), (4, 5), (6, 5), (7, 6), (8, 7), (9, 9)\}$$

$$a^*_4 = \{(1, 1), (2, 3), (3, 7), (4, 5), (6, 6), (7, 6), (8, 7), (9, 9)\}$$

$$a^*_5 = \{(1, 1), (2, 3), (3, 4), (4, 8), (5, 5), (6, 3), (7, 6), (8, 7), (9, 9)\}$$

The corresponding cost matrix is built as follows:

$$C_2 = \begin{pmatrix}
0.2 & 0.8 & 4 & \infty & \infty & \infty \\
0.86 & 0.23 & 4 & \infty & \infty & \infty \\
4.7 & 5.1 & \infty & \infty & \infty & 4 \\
10 & 10 & 0 & 0 & 0 & 0 \\
\infty & \infty & 0 & 0 & 0 & 0 \\
\end{pmatrix}$$

The corresponding 10-best assignments are:

$$a^*_1 = \{(1, 1), (2, 2), (3, 5), (4, 6), (5, 3), (6, 4)\}$$

$$a^*_2 = \{(1, 2), (2, 1), (3, 5), (4, 6), (5, 3), (6, 4)\}$$

$$a^*_3 = \{(1, 3), (2, 2), (3, 1), (4, 6), (5, 4), (6, 5)\}$$

$$a^*_4 = \{(1, 2), (2, 4), (3, 1), (4, 6), (5, 3), (6, 5)\}$$

$$a^*_5 = \{(1, 3), (2, 2), (3, 5), (4, 1), (5, 4), (6, 6)\}$$

$$a^*_6 = \{(1, 1), (2, 4), (3, 2), (4, 6), (5, 3), (6, 5)\}$$

$$a^*_7 = \{(1, 1), (2, 4), (3, 5), (4, 2), (5, 3), (6, 6)\}$$

$$a^*_8 = \{(1, 2), (2, 4), (3, 5), (4, 1), (5, 3), (6, 6)\}$$

$$a^*_9 = \{(1, 3), (2, 1), (3, 2), (4, 6), (5, 4), (6, 5)\}$$

$$a^*_{10} = \{(1, 3), (2, 1), (3, 5), (4, 2), (5, 4), (6, 6)\}$$

and the corresponding costs are 8.43, 9.66, 12.33, 12.9, 12.93, 13.1, 13.3, 13.5, 13.76 and 13.96, respectively. As shown in Table II, no repetition occurs in the track-to-measurement association within the top 10 best assignments. At the current scan, the problem of inefficient assignment generation is resolved. However, a large number of tentative tracks will appear in the coming scan because all the unassociated measurements are initialized as new tracks, which are tentative. These tentative tracks will also compete for data association diversity with other existing tracks. For example, assume that there are two confirmed tracks $T^1$ and $T^2$, two tentative tracks

<table>
<thead>
<tr>
<th>Track</th>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$T^{\text{new}}_1$</th>
<th>$T^{\text{new}}_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a^*_1$</td>
<td>$M_1$</td>
<td>$M_2$</td>
<td>$M_3$</td>
<td>$M_4$</td>
</tr>
<tr>
<td>$a^*_2$</td>
<td>$M_2$</td>
<td>$M_1$</td>
<td>$M_4$</td>
<td>$M_3$</td>
</tr>
<tr>
<td>$a^*_3$</td>
<td>$M_3$</td>
<td>$M_2$</td>
<td>$M_1$</td>
<td>$M_3$</td>
</tr>
<tr>
<td>$a^*_4$</td>
<td>$M_4$</td>
<td>$M_3$</td>
<td>$M_2$</td>
<td>$M_1$</td>
</tr>
<tr>
<td>$a^*_5$</td>
<td>$M_5$</td>
<td>$M_4$</td>
<td>$M_3$</td>
<td>$M_2$</td>
</tr>
<tr>
<td>$a^*_6$</td>
<td>$M_6$</td>
<td>$M_5$</td>
<td>$M_4$</td>
<td>$M_3$</td>
</tr>
<tr>
<td>$a^*_7$</td>
<td>$M_7$</td>
<td>$M_6$</td>
<td>$M_5$</td>
<td>$M_4$</td>
</tr>
<tr>
<td>$a^*_8$</td>
<td>$M_8$</td>
<td>$M_7$</td>
<td>$M_6$</td>
<td>$M_5$</td>
</tr>
<tr>
<td>$a^*_9$</td>
<td>$M_9$</td>
<td>$M_8$</td>
<td>$M_7$</td>
<td>$M_6$</td>
</tr>
<tr>
<td>$a^*_10$</td>
<td>$M_{10}$</td>
<td>$M_9$</td>
<td>$M_8$</td>
<td>$M_7$</td>
</tr>
</tbody>
</table>

Table II

Data Association for $C_2$ by using standard Murty’s algorithm.
1) **Stage 1:** Find the best assignment $a_1^*$ using optimal assignment algorithm like Auction algorithm [2] or JVC algorithm [9, 11].

$$a_1^* = \{(i_1, j_1), ..., (i_n, j_n)\}$$

Denote by $N_1, N_2, ..., N_{n-1}$ the nodes at stage 1.

$$N_1 = \{(i_1, \pi_1)\}$$

$$N_2 = \{(i_1, \pi_1), (i_2, \pi_2)\}$$

... 

$$N_r = \{(i_1, \pi_1), (i_2, \pi_2), ..., (i_r, \pi_r)\}$$

Then $\{N_1, ..., N_r\}$ forms a list of nodes for stage 1. Denote by $A$ the collection of all the assignments. Each node $N_i$ is a non-empty subset of $A$. All the nodes are mutually disjoint.

2) **General stage $k$:** Suppose $a_1^*, ..., a_k^*$ and $N_1, ..., N_k$ have been determined for this stage where $e$ denotes the number of current nodes. Denote by $a_i$ the best assignment in node $N_i$ and $Z(a_i)$ the corresponding cost, then the global $(k+1)$-th best assignment $a_{k+1}$ is given as

$$i^* = \arg \min_i Z(a_i),$$

$$a_{k+1}^* = a_{i^*}.$$ 

Assume $\varphi(k+1) = \{(i_1, \pi_1)\}$. Then nodes of partitioning $N_{i^*}$ by $\varphi(k+1)$ are given as

$$N_{i^*} = N_{i^*} \cup \{(i_1, \pi_1)\}$$

... 

$$N_{k+1} = N_{i^*} \cup \{(i_1, \pi_1), (i_2, \pi_2), ..., (i_{k+1}, \pi_{k+1})\}$$

where $\cup$ represents the union of two sets. Then the nodes for next stage are $\{N_1, ..., N_{i^*}, N_{i^*+1}, ..., N_{k+1}, ..., N_n\}$. The modified Murty’s algorithm reduces to the standard Murty’s algorithm when $r = n$.

### B. Example

In this subsection, the cost matrix $C^*$ in Section II-B is reused as an example to illustrate the modified Murty’s algorithm in detail. Recall that $C^*$ is given as follows:

$$
C^* = \begin{pmatrix}
T_1^0 & T_1^0 & T_2^0 & T_3^0 & T_4^0 & T_5^0 & T_6^0 & T_7^0 & T_8^0
\end{pmatrix}
\begin{array}{ccccccc}
M_1 & 0.30 & 4.70 & 4.92 & 4.91 & 5 & \infty & \infty & \infty & \infty
\end{array}
\begin{array}{ccccccc}
M_2 & 4.90 & 0.20 & 4.91 & 4.99 & 5 & \infty & \infty & \infty & \infty
\end{array}
\begin{array}{ccccccc}
M_3 & 4.10 & 4.90 & 4.89 & 4.93 & 5 & \infty & \infty & \infty & \infty
\end{array}
\begin{array}{ccccccc}
M_4 & 4.01 & 4.89 & 4.89 & 4.96 & 5 & \infty & \infty & \infty & \infty
\end{array}
\begin{array}{ccccccc}
M_5 & 4.20 & 4.79 & 4.87 & 4.97 & \infty & \infty & \infty & \infty & \infty
\end{array}
\begin{array}{ccccccc}
M_6 & 10 & \infty & \infty & \infty & \infty & 0 & 0 & 0 & 0
\end{array}
\begin{array}{ccccccc}
M_7 & \infty & 10 & \infty & \infty & \infty & 0 & 0 & 0 & 0
\end{array}
\begin{array}{ccccccc}
M_8 & \infty & \infty & 10 & \infty & \infty & 0 & 0 & 0 & 0
\end{array}
\begin{array}{ccccccc}
M_9 & \infty & \infty & \infty & 10 & \infty & 0 & 0 & 0 & 0
\end{array}
\begin{array}{ccccccc}
M_10 & \infty & \infty & \infty & \infty & \infty & 0 & 0 & 0 & 0
\end{array}
\end{pmatrix}
$$

Assume that the designated set of tracks are the set of confirmed tracks, i.e., $T_1^0$ and $T_2^0$. The goal is to find the top 5 different track-to-measurement associations for the set of designated tracks.

1) Denote by $N_0 = \emptyset$. By using the auction algorithm, the best assignment is given as

$$a_1^* = \{(1,1), (2,2), (3,4), (4,8), (5,3), (6,5), (7,6), (8,7), (9,9)\}$$

with cost $Z(a_1^*) = 15.3$. Let a subset of $a_1^*$ be $\varphi_1 = \{(i,j)\}$ such that $j$ belongs to the set of IDs of designated tracks. In this example, the set of IDs of designated tracks is $\{1,2\}$ and thus $\varphi_1 = \{(1,1), (2,2)\}$. Partition $N_0$ by $\varphi_1$, then the nodes for next stage are

$$N_1 = \{(1,1)\}$$
$$N_2 = \{(1,1), (2,2)\}$$

2) Denote by $a_i$ the optimal assignment within the current $i$-th node $N_i$. The second best assignment can be found as $a_2^* = \min_i a_i$, which is given as

$$a_2^* = \{(1,4), (2,2), (3,7), (4,1), (5,3), (6,5), (7,6), (8,9), (9,8)\}$$

with cost $Z(a_2^*) = 18.99$. The subset $\varphi_2 = \{(i,j)\}$ of $a_2^*$ such that $j \in \{1,2\}$ is $\varphi_2 = \{(2,2), (4,1)\}$. Since $a_2^*$ is in $N_1$, the new set of nodes is obtained by partitioning $N_1$ by $\varphi_2$ and keeping all the remaining nodes. The nodes for next stage are

$$N_1 = \{(1,1), (2,2)\}$$
$$N_2 = \{(1,1), (2,2)\}$$
$$N_3 = \{(1,1), (2,2), (4,1)\}$$

3) Based on current nodes, the third best assignment is

$$a_3^* = \{(1,4), (2,2), (3,1), (4,8), (5,3), (6,5), (7,6), (8,7), (9,9)\}$$

and $a_3^*$ is in $N_3$ and thus nodes for next stage are

$$N_1 = \{(1,1), (2,2)\}$$
$$N_2 = \{(1,1), (2,2)\}$$
$$N_3 = \{(1,1), (2,2), (4,1), (2,2)\}$$
$$N_4 = \{(1,1), (2,2), (4,1), (2,2), (3,1)\}$$

However, the set of valid assignments within node $N_3$ is empty. Therefore, this node is discarded and the remaining nodes for next stage are

$$N_1 = \{(1,1), (2,2)\}$$
$$N_2 = \{(1,1), (2,2)\}$$
$$N_3 = \{(1,1), (2,2), (4,1), (3,1)\}$$

4) Similarly, the fourth best assignment is

$$a_4^* = \{(1,4), (2,2), (3,3), (4,8), (5,1), (6,5), (7,6), (8,7), (9,9)\}$$
with cost $Z(a_4') = 19.40$. $\varphi_4$ is given as $\{(2, 2), (5, 1)\}$ and $a_4'$ is in $N_3$. The nodes for next stage are

\[
N_1 = \{(1, 1), (2, 2)\}, \quad N_2 = \{(1, 1), (2, 2)\}, \\
N_3 = \{(1, 1), (2, 2), (4, 1), (3, 1), (2, 2), (5, 1)\}, \\
N_4 = \{(1, 1), (2, 2), (4, 1), (3, 1), (5, 1)\},
\]

Similarly, node $N_3$ is empty. Therefore, node $N_3$ is removed and nodes for next stage are

\[
N_1 = \{(1, 1), (2, 2)\}, \quad N_2 = \{(1, 1), (2, 2)\}, \\
N_3 = \{(1, 1), (2, 2), (4, 1), (3, 1), (5, 1)\}.
\]

5) The fifth best assignment is

\[
a_5' = \{(1, 1), (2, 6), (3, 4), (4, 3), (5, 2), (6, 5), (7, 9)\}
\]

with cost $Z(a_5') = 20.21$. $\varphi_5$ is given as $\{(1, 1), (5, 2)\}$ and $a_5'$ is in $N_1$. Similarly, after removing the empty node, the remaining nodes for next stage are

\[
N_1 = \{(1, 1), (2, 2)\}, \quad N_2 = \{(1, 1), (2, 2)\}, \\
N_3 = \{(1, 1), (2, 2), (4, 1), (3, 1), (5, 1)\}.
\]

Since $M_9^g \sim M_9^g$ are dummy measurements, the track-to-measurement associations generated by the modified Murty’s algorithm are shown in Table IV. As shown in Table IV, there

<table>
<thead>
<tr>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$T_3$</th>
<th>$T_4$</th>
<th>$T_5$</th>
<th>$T_{\text{new}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>$M_1$</td>
<td>$M_2$</td>
<td>$M_5$</td>
<td>$M_3$</td>
<td>$M_4$</td>
</tr>
<tr>
<td>$a_2$</td>
<td>$M_1$</td>
<td>$M_2$</td>
<td>$M_5$</td>
<td>$M_1$</td>
<td>$M_3$</td>
</tr>
<tr>
<td>$a_3$</td>
<td>$M_1$</td>
<td>$M_2$</td>
<td>$M_5$</td>
<td>$M_1$</td>
<td>$M_4$</td>
</tr>
<tr>
<td>$a_4$</td>
<td>$M_1$</td>
<td>$M_2$</td>
<td>$M_5$</td>
<td>$M_1$</td>
<td>$M_3$</td>
</tr>
</tbody>
</table>

Table IV: Data association for $C^r$ by using the modified Murty’s algorithm.

is no repeated data association for the designated set of tracks.

IV. APPLICATION OF THE MODIFIED MURTY’S ALGORITHM IN MHT

A. Assignment generation with guaranteed diversity

In dense clutter scenarios, a large number of tentative tracks, which are very likely to be false tracks, will compete with the confirmed tracks for data association diversity. In addition, as discussed in Section II, the standard Murty’s algorithm cannot control the diversity competition among tracks and thus degrades the tracking performance in some scenarios. However, by using the modified Murty’s algorithm the assignment diversity can be controlled. Based on the modified Murty’s algorithm, the following algorithm can guarantee $K_c$ and $K_t$ assignment diversity, which is either specified by the user or selected adaptively, for both confirmed and tentative tracks, respectively. For every parent hypothesis $H(i)$, i.e., the hypothesis maintained at previous scan,

1) Set all the confirmed tracks as tracks of interest and find the $K_c$-best assignments by the modified Murty’s algorithm.
2) Set all the tentative tracks as tracks of interest and find the $K_t$-best assignments by the modified Murty’s algorithm.
3) Sort all these $K_c+K_t$ assignments by the corresponding costs and remove repeated assignments.

The final number of offspring hypotheses generated by this parent hypothesis $K_{p\rightarrow o}(i)$ is bounded as $\max(K_c, K_t) \leq K_{p\rightarrow o}(i) \leq (K_c+K_t)$. After exploring all the parent hypotheses, the top $K$ offspring hypotheses are selected and kept as the parent hypotheses for the next scan.

B. Adaptive selection of $K_c$ and $K_t$

As it has been proven in [7], the cost of each assignment is a negative logarithm of the probability of the corresponding hypothesis. In view of this, two heuristic rules are proposed for adaptive selection of $K_c$ and $K_t$ for every parent hypothesis $H(i)$. Assume that the maximum number of offspring hypotheses that can be spawned from a single parent hypothesis is $K_{p\rightarrow o}$, then

1) Select $K_c$ as the maximum integer such that 1) $K_c < K_{p\rightarrow o}$; 2) $CUM(K_c) \leq \beta \times p(H(i))$ where $CUM(K_c)$ is the cumulative probability of the top $K_c$ offspring hypotheses of parent hypothesis $H(i)$ and $p(H(i))$ is the probability of the parent hypothesis $H(i)$. The design parameter $\beta$ satisfies $0 < \beta \leq 1$.
2) Select $K_t$ such that the ratio $K_c : K_t$ equals to $Q_c : Q_t$ where $Q_c$ and $Q_t$ denote the total track quality [17] of confirmed tracks and that of tentative tracks, respectively. In practice, $K_t = \lceil \frac{Q_c}{Q_c+K_c} \rceil$.

The summary of the whole algorithm is given as follows:

- FOR every parent hypothesis $H(i)$
  1) Select $K_c+K_t$ offspring hypotheses based on the above mentioned two heuristic rules.
  2) IF $K_c+K_t > K_{p\rightarrow o}$
     a) Sort these $K_c+K_t$ offspring hypotheses based on their probabilities.
     b) Keep the top $K_{p\rightarrow o}$ offspring hypotheses and delete the others.
  3) END
  4) Add the current offspring hypotheses to the offspring hypotheses pool.
- END
- Sort all the offspring hypotheses in the pool.
- Keep the top $K$ offspring hypotheses from the pool and delete the others.

V. SIMULATION

A simulated tracking scenario is presented in this section to show the effectiveness of the proposed algorithm. The surveillance region is of size $2.0 \text{km} \times 5.0 \text{km}$. Three closely spaced
targets with separation distances 50m are moving in parallel for 14 scans with speeds 150m/scan. Then these targets take coordinate turn model and start to separate with different angular speeds $\frac{1}{25}\text{rad/scan}$, $\frac{1}{20}\text{rad/scan}$ and $\frac{1}{15}\text{rad/scan}$, respectively. The sensor measures the XY-positions of the targets with probability of detection $P_d = 0.95$ and false alarm density $\lambda_{F,A} = 0.2/\text{Km}^2$ and the standard deviation of the measurement errors in both X and Y direction are 50m. In addition, a rectangular high clutter area exists with boundaries $X_{\text{min}} = 1.5\text{Km}$, $X_{\text{max}} = 3\text{Km}$, $Y_{\text{min}} = 1\text{Km}$ and $Y_{\text{max}} = 2.5\text{Km}$, and clutter density in this region is $2/\text{Km}^2$. Two MHT trackers, one using the standard Murty’s assignment and the other using the modified assignment, are used to track the targets. In both MHT trackers, every parent hypothesis in previous scan will be expanded into at most $K_{p\rightarrow o} = 2$ offspring hypotheses in the current scan and only the top $K = 4$ current hypotheses will be maintained for next scan. In addition, the maximum target speed and the maximum angular speed are set to be $250\text{m/scan}$ and $\frac{1}{2}\text{rad/scan}$, respectively, and $\beta$ is 90%. Both trackers use Constant Velocity (CV) model with process noise $[2.5\text{m/scan}^2, 2.5\text{m/scan}^2]$ and Coordinate Turn (CT) models with process noise $[0.05\text{m/scan}^2, 0.05\text{m/scan}^2, 0.01\text{rad/scan}^2]$.

The tracking results of both trackers in a typical Monte Carlo run are shown in Figure 1–4. The tentative tracks are not shown in concern of the clearness of the figures. Track swap occurs for both trackers in their current best hypotheses at the 12-th scan as shown in Figure 1 and Figure 2. At the 17-th scan, the first tracker, which uses the standard Murty’s algorithm for data association generation, still gives twisted tracks for the two switched targets as shown in Figure 3 because the standard Murty’s algorithm cannot provide enough data association diversity for the confirmed tracks and the correct assignment is outside the top $K$ ($K = 4$ in this simulation) assignment queue and thus discarded. In addition, this unresolved swap messes later data associations and thus gives twisted tracks in later scans. However, the second tracker, which uses the proposed algorithm, is able to maintain greater diversity of the track-to-measurement association for the confirmed tracks within the top $K$ assignments. Consequently, in the second tracker the correct assignment, which is not the dominant one at previous scans, is kept within the top $K$ assignments and then becomes dominant as time evolves to later scan. Therefore, the second tracker is able to provide cleaner trajectories for these three targets as shown in Figure 4.

In addition, Table V summarizes the results of these two MHT trackers averaged over 100 Monte Carlo runs. It should be admitted that the improvement of the RMSE performance of the MHT using the proposed algorithm, as compared to the MHT using the standard Murty’s algorithm, is not significant ($\approx 1.2\%$) because the target separations are small as compared to the measurement noise, i.e., the targets are closely spaced. However, the MHT using the proposed algorithm, as compared to the MHT using the standard Murty’s algorithm, reduces the average number of swaps [16] $10.7\%$, and increases the track continuity [3] from $71.36\%$ to $74.50\%$ as shown in Table V.

<table>
<thead>
<tr>
<th>Performance metric</th>
<th>MHT tracker using the standard Murty’s algorithm</th>
<th>MHT tracker using the modified Murty’s algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE (Position)</td>
<td>$21.58(\text{m})$</td>
<td>$21.26(\text{m})$</td>
</tr>
<tr>
<td>Average number of swaps in tracks</td>
<td>$1.77$</td>
<td>$1.68$</td>
</tr>
<tr>
<td>Track continuity</td>
<td>$71.36%$</td>
<td>$74.50%$</td>
</tr>
</tbody>
</table>

Table V

**Performance comparison of MHT trackers using standard and modified Murty’s algorithm over 100 Monte Carlo runs.**

**VI. Conclusion**

In this paper, a modified Murty’s algorithm that provides more efficient and flexible target-to-measurement assignments was proposed. Unlike the standard Murty’s algorithm, which has no control over the diversity of target-to-measurement assignments, the modified Murty’s algorithm improves the track continuity and lowers the number of swaps in the association process.
associations and generates only slightly varied top K assignments, the modified Murty’s algorithm controls the data association diversities of different types of tracks by assigning them different priorities during the partition procedure. The result is that the data association diversity of a certain set of designated tracks, e.g., confirmed tracks or any user-specified tracks, is ensured in the top K assignments and thus improves the tracking performance. In addition, taking advantage of the controllability of data association diversity among different types of tracks in the modified Murty’s algorithm, a heuristic algorithm, which is capable of adaptively allocate priority and thus provide reasonable diversities to different types of tracks according to their track qualities, was proposed in this paper. Moreover, the numerical examples show that, by using the modified Murty’s algorithm, the tracker is able to maintain a more efficient set of hypotheses in complicated tracking scenarios and thus provides better tracking performance than the tracker that uses the standard Murty’s algorithm.

APPENDIX

Assume that the the original target-to-measurement association cost matrix is given as

\[
C_3 = \begin{pmatrix}
T_1 & T_2 \\
M_1 & 0.2 & 0.8 \\
M_2 & 0.86 & 0.23 \\
M_3 & 4.1 & 4.9 \\
M_4 & 4.7 & 5.1 \\
M_5 & 4.2 & 4.9 \\
M_6 & 4.5 & 5.2 \\
M_7 & 4.2 & 4.3 \\
\end{pmatrix}
\]

The 10-best assignments generated by the standard Murty’s algorithm are

\[
\begin{align*}
\alpha_1^* &= \{(1, 1), (2, 2), (3, 12), (4, 13), (5, 14), (6, 15), (7, 16)\} \\
\alpha_2^* &= \{(1, 1), (2, 2), (3, 5), (4, 13), (5, 14), (6, 15), (7, 16)\} \\
\alpha_3^* &= \{(1, 1), (2, 2), (3, 12), (4, 6), (5, 14), (6, 15), (7, 16)\} \\
\alpha_4^* &= \{(1, 1), (2, 2), (3, 12), (4, 13), (5, 7), (6, 15), (7, 16)\} \\
\alpha_5^* &= \{(1, 1), (2, 2), (3, 12), (4, 13), (5, 14), (6, 8), (7, 16)\} \\
\alpha_6^* &= \{(1, 1), (2, 2), (3, 12), (4, 13), (5, 14), (6, 15), (7, 9)\} \\
\alpha_7^* &= \{(1, 1), (2, 2), (3, 5), (4, 6), (5, 14), (6, 15), (7, 16)\} \\
\alpha_8^* &= \{(1, 1), (2, 2), (3, 5), (4, 13), (5, 7), (6, 15), (7, 16)\} \\
\alpha_9^* &= \{(1, 1), (2, 2), (3, 5), (4, 13), (5, 14), (6, 8), (7, 16)\} \\
\alpha_{10}^* &= \{(1, 1), (2, 2), (3, 5), (4, 13), (5, 14), (6, 15), (7, 9)\}
\end{align*}
\]

The track-to-measurement associations are shown in Table VI. The data association for the two confirmed tracks are exactly the same, which is very inefficient.

<table>
<thead>
<tr>
<th>Track</th>
<th>Measurement</th>
<th>( T_1 )</th>
<th>( T_2 )</th>
<th>( T_3^{\text{new}} )</th>
<th>( T_4^{\text{new}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>a_1^*</td>
<td>M_1</td>
<td>( \backslash )</td>
<td>( \backslash )</td>
<td>( \backslash )</td>
<td>( \backslash )</td>
</tr>
<tr>
<td>a_2^*</td>
<td>M_1</td>
<td>M_2</td>
<td>( \backslash )</td>
<td>( \backslash )</td>
<td>( \backslash )</td>
</tr>
<tr>
<td>a_3^*</td>
<td>M_1</td>
<td>M_2</td>
<td>M_4</td>
<td>( \backslash )</td>
<td>( \backslash )</td>
</tr>
<tr>
<td>\alpha_4^*</td>
<td>M_1</td>
<td>M_2</td>
<td>M_4</td>
<td>M_5</td>
<td>( \backslash )</td>
</tr>
<tr>
<td>\alpha_5^*</td>
<td>M_1</td>
<td>M_2</td>
<td>M_4</td>
<td>M_5</td>
<td>M_6</td>
</tr>
<tr>
<td>\alpha_6^*</td>
<td>M_1</td>
<td>M_2</td>
<td>M_4</td>
<td>M_5</td>
<td>M_6</td>
</tr>
<tr>
<td>a_7^*</td>
<td>M_1</td>
<td>M_2</td>
<td>M_6</td>
<td>( \backslash )</td>
<td>( \backslash )</td>
</tr>
<tr>
<td>\alpha_8^*</td>
<td>M_1</td>
<td>M_2</td>
<td>M_6</td>
<td>M_7</td>
<td>( \backslash )</td>
</tr>
<tr>
<td>\alpha_9^*</td>
<td>M_1</td>
<td>M_2</td>
<td>M_6</td>
<td>M_7</td>
<td>M_8</td>
</tr>
<tr>
<td>\alpha_{10}^*</td>
<td>M_1</td>
<td>M_2</td>
<td>M_6</td>
<td>M_7</td>
<td>M_8</td>
</tr>
</tbody>
</table>

Table VI

DATA ASSOCIATION FOR \( C_3 \) BY USING THE STANDARD MURTY’S ALGORITHM.

REFERENCES


