Navigation in GPS-Denied Environments

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Abstract—The Theater Positioning System (TPS), which can perform in GPS-denied environments and can work with, or independently of, GPS systems, is used as a backup to GPS in military. The principal difficulty in optimally combining this new system and GPS is caused by the somewhat unpredictable signal propagation of the TPS groundwave signal and thus results in less accurate performance when TPS works unaided in the environment while GPS is unavailable. A new navigation scheme that can provide an accurate position estimation for the GPS-denied user is developed. The scheme adopts a state-space model which is represented by stochastic differential equations (SDEs) and is used to predict the those propagation disturbances. We also propose a stochastic approximation method to solve the pseudorange equations which does not require prior knowledge of the noise covariance matrices. A numerical example is provided and compared to illustrate the performance of the methods proposed in this paper.

I. INTRODUCTION

An integrated radio-navigation system is introduced for individuals and assets which will perform in GPS-denied environments, including canyons, urban areas, underground, and deep inside buildings. The foundation of this system is the reception of appropriate 90-110 kHz ground-wave radio-frequency (RF) location signals via the newly developed Theater Positioning System (TPS) [1], which can be combined with GPS as a navigation backup for GPS. Like GPS, this enhanced LF component of the system uses spread-spectrum transmission for improved accuracy, exhibits a large processing gain for greater interference immunity, and thus has a significant advantage over conventional LORAN-C radio-navigation systems.

The predecessor of the TPS component of the navigation system is LORAN, which was the original very wide-area radio-navigation system that preceded GPS. Its relatively long wavelength (3000 meters) provided large geographic coverage via ground-wave propagation without the need for satellites. Furthermore, its long wavelength provides significant immunity from false locations due to local multipath. However, conventional LORAN is limited by lack of resolution and is highly susceptible to interference. Incorporating spread-spectrum signals at LORAN frequencies provides improved location precision and better interference immunity.

In the usual operating mode, GPS serves as the principal positioning source. Continuity of their fixes are assured, since during the normal TPS tracking process, the TPS and GPS position data are continually compared. As long as the recent and current GPS signal quality is good, the displayed TPS fix will be automatically adjusted to overlay the GPS values; this is generally done to provide an ongoing in-situ calibration of the TPS signal propagation delay figures and thus “drag” the TPS fix in to match the GPS. If GPS suddenly fails to provide a clean or continuous fix, the TPS value will track the last good GPS coordinates. Once GPS signal integrity is restored for at least a few seconds and a new lock is satisfactorily obtained, the system will smoothly revert to the GPS fix and return to normal operation. In the event that GPS is jammed or otherwise unavailable for an extended period, TPS will be employed in a standalone mode to derive the unit’s fix, with caution to the user that fix accuracies may be reduced. Another specific advantage of the TPS concept lies in the use of TPS as an antispoofing detector for GPS. For instance, if the TPS (presumed stable) and GPS planar fixes do not essentially coincide (i.e., where the GPS solution is considerably off from the TPS fix), this could be an indicator of GPS receiver problems or of the presence of a spoofing signal.

As mentioned in the previous paragraph, the fix accuracies are reduced when TPS works alone. The transmission errors \( (\eta_i) \) in section III-B) in the position estimation for TPS transmissions are caused by environmental factors such as the earth surface underlying the propagation path (e.g., water or land), as well as local variations in the surface types (e.g., terrain, soil types, vegetation). Unlike errors in GPS such as ionospheric and tropospheric delays which have known models [2], [3], the errors in TPS are harder to capture and difficult to be approximated by an exact model. The positioning accuracy thus degrades when TPS is used without special calibration factors (similar to those historically used in LORAN). In this work, we will propose a state-space model which captures the errors dynamically, and thus can guarantee the accuracies of TPS.

We also present an algorithm used to solve the pseudorange equations in both GPS and TPS. Most of the techniques presented in the literature have applied Newton-Raphson [2], [3], Kalman filter [4]–[6] or particle filter [7] methods to estimate the fixes. However, all of these algorithms require known noise matrices (Kalman filter and particle filter) or else do not take these noise components into consideration (Newton-Raphson). In this work, we will propose a different algorithm that does not need any specific information on the noise variances but can still calculate the user’s position.
efficiently.

The contributions of this paper are summarized as follows:

• A dynamic state-space model based on SDEs is introduced to model those errors and predict the future errors in TPS when GPS is denied;
• A stochastic approximation (SA) approach is proposed to compute the position explicitly; and
• A navigation scheme is put forward and demonstrated through the proposed error model and SA algorithm.

The paper is organized as follows: in section II, we discuss TPS in detail. In section III, navigation equations for GPS and TPS are given, plus an introduction to the SA algorithm used in this paper. In section IV, the detailed SA algorithm is proposed. In section V, the state-space model is illustrated and the navigation scheme is provided in section VI. Finally, in section VII, an example is given to show the performance of the method in the work.

II. TPS CONCEPT

A major operational concern for deploying U.S. military and law-enforcement personnel is the nearly exclusive dependence on the GPS satellite constellation for accurate position information in the field. Since GPS signals are comparatively weak (10-15 dB below the typical background RF noise floor) and subject to significant degradation from multipath and RF interference (intentional and unintentional), the use of GPS is at times unreliable and even subject to deception (“spoofing”) by an adversary [1]. The obvious consequences of inaccurate position information can be severe, up to and including loss of life of both friendly forces and/or noncombatants (civilians). Although inertial navigation systems (INS) have been proposed as short-term backups to GPS reception during outage periods, these units are in general too costly, heavy, bulky, inaccurate, and/or power-hungry to be deployed except in a few specialized applications. For dismounted personnel and most platforms, a much more robust, inexpensive, and reliable GPS augmentation technique is needed. For broad application areas, an RF approach is desirable; in addition, the use of ground-wave LF transmissions is radically different in propagation characteristics from GPS and thus provides a significant measure of signal diversity between the two radiolocation schemes. In addition, the use of the Theater Positioning System (TPS) [1] signals as a backup to GPS navigation offers far more consistent coverage than with GPS alone, since the low-frequency TPS signals can easily penetrate into most buildings, heavy foliage, urban terrain, and other areas where GPS signals are weak, unreliable, or even receivable; such expanded coverage is essential for successful operations in urban areas, very rough terrain, or in tropical or heavily forested regions. Furthermore, due to the extremely low signal strengths of the GPS satellite beacon transmitters at the receiver, GPS signals are very often unusable indoors because of the additional attenuation of the overhead satellite signals by building roofs, upper floors, and other overhead structures, as well as trees and dense foliage in general. In addition, in “urban canyons” and in very rugged terrain, often there are too few GPS satellites in direct line-of-sight view of the receiver to obtain a sufficiently accurate position fix. Again, TPS provides a much-needed improvement in locating-system reliability.

The basic configuration of the TPS scheme is shown in Fig. 1 below. The spread-spectrum (SS) TPS signals originate from multiple widely-spaced, terrestrial transmitters. The radiolocating receiver acquires these transmitted signals and extracts the transmitter locations and times of transmission from data streams embedded in the respective SS signals, in a manner analogous to GPS units. The radionavigation solutions are then obtained by solving the usual systems of nonlinear pseudorange equations by linearization techniques, Newton-Raphson estimation, Kalman filtering, particle filtering, multidimensional kernels or other means, but with downstream corrections for the spherical-earth geometry and RF propagation factors governing the ground wave signals. However, there are several significant features of the TPS which differentiate it from GPS, including its operating frequency range (100 kHz), propagation modes, and signal security mechanisms.

TPS transmitters are typically, although not necessarily, deployed outside the main area of operations, in a reasonably regularly spaced array to provide favorable angles of reception from the various transmitter locations (i.e., “good system geometry” or “cuts”). The mathematical equations used to calculate the respective ranges from the TPS receiver to the transmitters in the area (which could easily exceed 1000 km in distance), called the pseudorange equations in GPS parlance, are similar to the GPS versions, except that the TPS transmissions are generally from stationary sources and, as such, do not need Doppler or relativistic corrections to the pseudorange values before computing the location solution in the receiver. They do, however, require great-circle distance corrections for the ground-wave signal paths on the earth’s surface and adjustments to the propagation velocity values over the intervening terrain due to changes in the dielectric constant from varying soils, moisture content, etc. Like GPS, the TPS setup utilizes a precise common time base to provide highly accurate, stable time-of-day information for each transmitter; as in GPS, a stable clock in the TPS receiver permits faster initial signal acquisition and more accurate positioning via algorithms which incorporate strategic averaging among the various TPS signals.

III. NAVIGATION EQUATIONS

In this paper, we consider the navigation of the users on the surface of the earth that are subject to environmental conditions such as urban areas, very tough terrain, or in tropical or heavily forested regions. The calculation of the distance between the user and TPS transmitters should accommodate the ground-wave propagation and great-circle path distances; this is done by adjusting the equivalent speed of the wave for the slower propagation along the earth’s surface; the curved-path distances may then converted to the equivalent chord distances to utilize the normal rectilinear distance equations.
In the following subsections, we first discuss the basic GPS pseudorange equation, then the corrected great-circle distance equation, after which an SA method is proposed to solve those pseudorange equations.

A. GPS Pseudorange Equation

The principle of GPS navigation can be represented as follows [2], [3]: each satellite is sending out signals with the following content: I am satellite X, my position is Y and this information was sent at time Z. These orbital data (ephemeris and almanac data) are stored by the GPS receiver for later calculations. For the determination of its position, the GPS receiver compares the time when the signal was sent by the satellite with the time the signal was received. From this time difference the distance between receiver and satellite can be calculated. If data from other satellites are taken into account, the present position can be calculated by trilateration (the determination of a distance from three points). This means that at least three satellites are required to determine the position of the GPS receiver on the earth’s surface. The calculation of a position from 3 satellite signals is called a 2D-position fix (two-dimensional position determination); it is only two-dimensional because the receiver has to assume that it is located on the earth’s surface. By using four or more satellites, an absolute position in a three-dimensional space can be determined. A 3D-position fix also gives the height above the earth surface as a result. The pseudorange of the ith transmitter at time k is given by the equation:

\[ \rho_k = \rho_k^T + c(\zeta_k^T - \zeta_k^R) \]  

where \( \rho_k \) is the pseudorange computed by the time difference between the receiver and the ith satellite and \( \rho_k^T \) is the real range from the user to GPS satellite at time k; c is the velocity of the transmission signal. The pseudorange contains two primary sources of error. One error is introduced by the receiver’s clock, which is denoted as \( \zeta_k^R \) and called the receiver clock offset. This error remains the same in each pseudorange equation of each transmitter at time k. The other error is introduced in the transmission of GPS signal and denoted as \( \zeta_k^T \). This error can be modeled and approximated accurately [3], and thus is assumed known to the users. \( \rho_k^T \) denotes the true distance between the user and ith GPS satellite at time k. If we denote the ith satellite position by \( (X^i, Y^i, Z^i) \) relative to the center of the earth in Earth-Centered, Earth-Fixed (ECEF) coordinates, and the user’s position by \( (X_k, Y_k, Z_k) \) in the same coordinates, then the distance between the ith satellite and the user can be written as the non-linear expression:

\[ \rho_k^T = \sqrt{(X_k - X^i)^2 + (Y_k - Y^i)^2 + (Z_k - Z^i)^2} \]  

To solve the user positions and receiver clock offset, 4 satellites are needed to solve \( (X_k, Y_k, Z_k, \zeta_k^R) \) sufficiently.

B. TPS Great-Circle Distance

As mentioned above, the multilateration radiolocation algorithms for TPS are generally similar to those used in GPS except for the addition of great-circle corrections to accurately represent the lengths of the ground-wave propagation paths on the nearly spherical earth and (obviously) the deletion of the satellite almanac and ephemeris data. In most operational scenarios, the TPS transmitters will be locked to GPS time with very high-quality clocks; plus, their locations will be pre-surveyed and will be known to fractions of a meter. The respective TPS data streams will thus provide all the information needed by the receiver (except for onboard-stored local propagation-correction tables) to accurately compute its position. Due to the finite conductivity of the earth’s surface, and local variations due to surface types (i.e., land or water), soil, moisture content, temperature, and (to a lesser extent) season, the average signal velocity must be reduced by very roughly 0.15%. In addition, the curved path on the earth’s surface requires generic great-circle distance computations. As shown in Fig. 2, the true range transmitted is along the spherical earth instead of the chord between A (the user) and T (the transmitter) and should be estimated by the great-circle distance.

The TPS ground wave follows the great-circle distance between two points on the earth’s surface (assumed spherical), which can be computed by the following formula, where \( \delta^i \) and \( \varphi^i \) are latitude and longitude, respectively and r is the radius of the earth (approximately 6371 km on average), then
the great-circle distance \( d \) is approximately:

\[
d(\delta^1, \varphi^1, \delta^2, \varphi^2) = r \cos^{-1}[\sin \delta^1 \sin \delta^2 + \cos \delta^1 \cos \delta^2 \cos (\varphi^1 - \varphi^2)]
\] (3)

The great-circle distance equation is employed to calculate the distance of a near-spherical earth path between the user and land-based TPS transmitters. In this paper, we consider only the navigation of the users near the surface, which means the height between the user and the earth surface is zero. For users at varying heights, the distances between the users and TPS transmitters do not quite follow the great-circle equations and should be calculated by taking the heights of the users into account, which is out of the scope of this work.

Now assume there are \( M \) TPS transmitters. Then, the pseudorange equation at time \( k \) for TPS can be written similarly as that of GPS as follows:

\[
d_k = d_k^i + c_T(\eta_k^i - \eta_k^R)
\] (4)

where \( d_k^i \) is the pseudorange between the user and the \( i \)th TPS transmitter, and \( d_k^R \) is the true range between the user and the \( i \)th (1 \( \leq i \leq M \)) TPS transmitter, which is approximated by the great-circle equation given above; \( c_T = (1 - 15\%)c \) is the velocity of TPS transmission signal. \( \eta_k^i \) is the transmission error generated in the transmission of TPS signal by the environment around the surface and is what we need to model. \( \eta_k^R \) is the receiver clock offset, equivalent to \( \zeta_k^R \) in the GPS pseudorange equation. \( \eta_k^R \) is unfortunately difficult to model accurately due to its characteristic irregularity. When GPS is available, we can calculate \( \eta_k^R \), but when GPS is absent, we do not have enough information to do so. This then motivates us to find out a model that can be used to simulate and predict it.

### IV. Stochastic Approximation Method

To solve the pseudorange equations explicitly, numerous algorithms have already been proposed in the existing literature such as the Kalman Filter, Newton-Raphson method, particle filter, and the like [2]–[7]. However, most of them require either the variances of the noise (Kalman Filter and particle filter), or do not consider the effect of the noise (e.g. Newton-Raphson). In this section, a stochastic approximation algorithm in [8] is employed to compute the fixes explicitly. This method trains the Kalman gain matrix to its correct, steady-state form, in which sense the plant noise and observation noise covariance matrices are unknown. This SA algorithm can be concluded from [8] and is stated as follows: Consider the discrete-time system of user dynamics [9]:

\[
x_{k+1} = Ax_k + w_k
\] (5)

\[
y_k = d_k(x_k) + v_k
\] (6)

where \( x_k \) is the state vector containing the longitude, latitude (or X, Y, Z fixes in ECEF coordinates) of the user at time \( k \), velocities, and clock offsets; \( A \) is the corresponding system matrix; \( d_k(x_k) \) is the pseudorange vector containing all the pseudoranges [as in (1,4)] sampled for each TPS transmitter and is a function of \( x_k \); and \( w_k \) and \( v_k \) are uncertainties of the system.

It is well known from Kalman filter theory that the posterior estimate of \( x_k \) is given by:

\[
\hat{x}_{k+1} = A\hat{x}_k + K_{k+1}(y_{k+1} - d_k(A\hat{x}_k))
\] (7)

Instead of the traditional Kalman gain, the stochastic approximation provides a recursive gain adaptation algorithm in the form:

\[
K_{k+1} = K_k + \mu_k\theta(K_k)
\] (8)

where \( \mu_k \) is a decreasing sequence of real numbers and \( \theta(K_k) \) is an unspecified stochastic vector that depends on \( K_k \). One choice for \( \theta(K_k) \) is \( \theta(K_k) = A\hat{x}_{k+1}v_{k+1}^T \), and under certain conditions of \( \mu_k \) in [8], \( K_{k+1} \) converges to the optimal Kalman gain.

The advantages of this SA algorithm over other algorithms are summarized as follows:

- It does not need the information from noise covariance matrices;
- The computation of its Kalman gain does not require the calculation of the estimation covariance, which can reduce the computation cost significantly over that of the Kalman filter.
- Unlike Newton-Raphson, which needs \( N \) equations to solve \( N \) variables, SA can estimate \( x_k \) completely with a number of measurements smaller than the number of variables contained in the state \( x_k \) (partially observed).

### V. State Space Model

In this section, we now model the errors produced during the transmission of TPS via a dynamic state-space model which is based on SDEs. SDEs have been widely used previously to model control systems and communication channels. For example, a mobile-to-mobile communication channel can be modeled as [10]:

\[
x_{k+1} = F_kx_k + G_kw_k
\]

\[
y_k = H_kx_k + N_kv_k
\] (9)

where \( x_k \) is the state; \( y_k \) is the measurement sampled at the output of the channel; \( w_k \) and \( v_k \) are noises; and \( F_k, G_k, H_k, N_k \) are parameters of the channel.

Other related examples can be found in [11]–[14], where the great flexibility and utility offered by state-space models are employed to generate extensive applications in a number of different areas of statistics. For example, the Bureau of Labor Statistics (BLS) in the U.S. uses state-space models for the estimation of all the monthly employment and unemployment estimates for the 50 states and the District of Columbia; here, the state-space models are fitted independently between the internal states and used to build a model of the true population values with an accompanying model for the sampling errors [11]. The authors in [12] use the state-space approach to model, estimate, and predict short-term electric power consumption, which can provide an insight into future power demand and thus make decisions on the power generation,
e.g., whether to activate reserve power generators or decrease generator outputs. [13] generalized the linear time-discrete state-space model from a single-dimensional time to two-dimensional space and then used it as a model for linear image processing. Further, [14] developed a non-Gaussian state-space model for censored data. These applications suggest its successful use in modeling and predicting TPS transmission errors.

The time-varying property of the parameters in (9) adapts dynamically to the variety of states. The noises \( w_k \) and \( v_k \) can also capture the range uncertainties introduced during the transmission. Due to the these special characteristics, we propose to use the state-space model to track, estimate and predict the stochastic behavior of the transmission errors (\( \eta_k \)) in TPS transmissions. Consider the following time-invariant state-space predictor:

\[
x_{k+1} = F x_k + G e_k \\
\eta_k = H x_k + e_k
\]

where \( \eta_k \) is the transmission error computed for each transmitter, and \( e_k = \eta_k - H x_k \) is the error between the measured error and the error predicted by the model. To estimate the parameters \( F, G, H, \) and \( x_k \), the prediction error minimization (PEM) method [15] is employed. PEM estimates the parameters by minimizing a least-square cost function:

\[
V_N = \sum_{k=1}^{N} e_k^T e_k
\]

The details of the algorithm can be found in [15].

After estimating the parameters \( F, G, H \) and \( x_k \) at time \( k \), PEM can then update them with the new measurement at time \( k+1 \). With the estimated parameters and states of the model, the one step-ahead prediction of the transmission error can be computed by:

\[
\hat{\eta}_{k+1} = \hat{H}_k (\hat{F}_k \hat{x}_k + \hat{G}_k (\eta_k - \hat{H}_k \hat{x}_k))
\]

where \( \hat{\eta}_{k+1} \) denotes the predicted transmission error at \( k+1 \); \( \hat{F}_k, \hat{G}_k, \hat{H}_k \) and \( \hat{x}_k \) are the parameters and state estimated by PEM at time \( k \). The \( p \)-steps-ahead (\( p > 1 \)) prediction of the transmission error can be computed as:

\[
\hat{\eta}_{k+p} = \hat{H}_k \times \hat{F}_k^{p-1} (\hat{F}_k \hat{x}_k + \hat{G}_k (\eta_k - \hat{H}_k \hat{x}_k))
\]

VI. NAVIGATION SCHEME OF TPS

When GPS signal is available, the user’s position is sufficient to be calculated by the SA method in section IV. Meanwhile, the TPS algorithm can also compute the position through great-circle equations by generally similar techniques. Note that there is no exact model for the errors introduced in TPS signal transmission and the range transmitted is approximated by the great-circle distance equation; thus the position estimated via TPS is not as close to the actual position estimate as that via GPS. The problem is: when GPS is not available due to environmental factors, how do we navigate using TPS with good accuracy while referencing to a batch of old GPS data? In this section, we will now introduce the following navigation scheme to improve the accuracy.

**Navigation Scheme Algorithm (NSA):** Assume GPS and TPS data are available from time 1 to time \( n \). Also from time \( n \) on, only TPS data is accessible. The procedure to navigate with TPS only at time \( n + 1 \) is as follows:

1) Compute the user’s position \((X^k, Y^k, Z^k)\) and \( \zeta^k_R \) for each \( k = 1, \ldots, n \) using (1) and (2) through the SA algorithm. Then convert \((X^k, Y^k, Z^k)\) to latitude and longitude. As the user is assumed near the earth’s surface, the height of the user \( H \) is 0.

2) Plug the latitude and longitude back into (3) to obtain \( d_{LR}^k \) terms for each \( k = 1, \ldots, n \) and each transmitter.

3) As the receiver clock offset is constant in one single time slot, we can assume that \( \eta_{LR}^k = \zeta^k_R \). Then \( \eta_{LR}^k \) can be computed through (4) for each \( k \) and each \( i \) (Note \( d_{LR}^k \) are measured by TPS).

4) Build a state-space model for each \( \{\eta_{LR}^k\}_{k=1}^{n} \) using the technique in section V and predict \( \eta_{LR}^{n+1} = C_n \tilde{x}_{n+1} \) for each transmitter, where \( \tilde{x}_{n+1} \) is the predicted state from the state-space model.

5) Plug \( \eta_{LR}^{n+1} \) back into (4) and obtain the measurement equations. Together with the system equation (5), the state \( x_{n+1} \) can now be estimated sufficiently by the SA algorithm.

After time \( n + 1 \), continue to compute the user’s positions with the previous algorithm when the GPS signal is lost. Once GPS becomes available again, update the state-space model with the new \( \eta^k \) values computed from the latest GPS measurement.

**Remark:** For the users with varying heights (\( H \neq 0 \)), the distance equation to compute the true range should be updated. However, it is difficult to determine a unique equation in this case, as the user may below the average earth altitude (e.g., a canyon) or on a hill, where the equations are different (see Fig. 3). In a practical situation, the range may be approximated by the great-circle equation. However, we maintain that once the distance equation is altered for varying heights, the navigation scheme proposed in this paper is still applicable to the new distance equation.
VII. NUMERICAL EXAMPLE

In this section, we now offer an example to illustrate the performance of the navigation algorithm proposed in this paper. The simulation result is based on MATLAB.

Assume N=3 TPS transmitters are located with latitude and longitude pairs: (38.3127491°, 115.6442846°), (39.2763475°, 116.0855268°), (37.6413982°, 114.3172851°). The initial position of the user in ECEF coordinates is \((-2.172 \times 10^6, 4.390 \times 10^6, 4.074 \times 10^6)\).

The user is assumed moving along the earth’s surface randomly. Thus, for convenience but without loss of generality, the distance equation can be written as (5) and (6), where

\[ A = I_{4 \times 4} \]

the state vector \( x_k^T = [\delta_k, \varphi_k, \eta_{1k}^R] \) for TPS and \( x_k^G = [X_k, Y_k, Z_k, \zeta_{1k}^R] \) for GPS.

From time 1 to 50, when both GPS and TPS data are available, \( \{x_k^G\}_{k=1}^{50} \) are computed by the SA algorithm and then \( \{\eta_k^R\}_{k=1}^{50} \) can be obtained by following NSA in section VI. From times 51 to 150, GPS is denied and only TPS is available. A scalar state-space model is employed to model \( \{\eta_k^i\}_{k=51}^{150} \), and then \( \{\hat{\eta}_k^i\}_{k=51}^{150} \) are predicted by this model through the algorithm proposed in section V. After that, the positions of the user are estimated through the SA algorithm during times 51 to 150. The results are presented in Figs. 4-6 (shown in ECEF coordinates for the sake of comparison). It is obvious that the positions estimated by the proposed navigation scheme and algorithm are close to the actual values since the differences between the estimated and the true fixes are small. The percentage of the error between actual \( \{\eta_k^i\} \) and predicted ones \( \{\hat{\eta}_k^i\} \) \( (|\eta_k^i - \hat{\eta}_k^i|/\eta_k^i) \) by the state-space model are also plotted in Fig. 7-Fig. 9. These plots demonstrate that the proposed model can predict the transmission errors with small differences.

VIII. CONCLUSION AND FUTURE WORK

In this paper we studied the navigation procedure in the Theater Positioning System (TPS), which is largely intended to be used as a backup in GPS-denied environments. We have considered the user moving along the earth’s surface and have...
employed a state-space model method to predict the error generated by environmental delays in its transmission, thus improving the estimation accuracy of TPS fixes. We have also proposed a stochastic approximation algorithm to solve the pseudorange equations. Lastly, an example was provided to demonstrate that the estimation performance of the algorithm is quite satisfactory.

REFERENCES