Target Tracking in Uncertain Multipath Environments using Viterbi Data Association

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Abstract – This paper addresses the problem of accurate geolocation of targets or emitters in an uncertain multipath propagation channel. We propose a new approach which exploits differences between target and multipath dynamics to jointly estimate target position and channel parameters. The concept is derived here for an uncertain indoor propagation channel where multipath specular reflections from walls can be adequately captured using ray-trace models. A multi-mode Viterbi data association algorithm (MM-VDA) is presented which determines the unknown correspondences between measurements and multipath propagation modes in the presence of false alarms. The method is evaluated based on simulations where a moving radio frequency (RF) emitter is tracked in an uncertain multipath indoor channel even when the direct-path return is occasionally not present.

Keywords: Target geolocation, multipath channel estimation, Viterbi data association

1 Introduction

Target geolocation in multipath environments is often degraded due to the difficulty in correctly associating non-line-of-sight (NLOS) returns caused by obscuration by terrain, buildings and other structures [1]. In such cases, prior information of the multipath propagation channels is often required to give an accurate estimate of target location [2, 3]. Alternatively, the estimation of the dynamic target position and mode parameters can be treated as correlated processes and estimated jointly as in [5]. Joint estimation of target position and multipath parameters is similar in concept to approaches for simultaneous localization and mapping (SLAM) developed for robotics [4]. In SLAM techniques, mapping of the unknown environment is achieved while updating the location or pose of the moving sensor platform.

In this paper, we consider a general framework for estimating a state vector which contains both target position and multipath modes parameters. Two requirements for this approach are: 1) a stochastic model for the dynamics of the targets and multipath channels parameters, and 2) a set of observations (e.g. time-delays and bearing angles) which over time can be used to infer the underlying state sequence. Although we do not prove observability in this paper, we do offer a simulation example where multipath parameters are inferred from one-way measurements of time-delays and bearings. In particular, we consider an indoor tracking problem where the uncertain multipath channel is a result of specular reflections off interior building walls. An important challenge addressed here is the problem of associating the measurements and the propagation modes that produced them in the presence of clutter and noise. In this paper, Viterbi Data Association (VDA) [7-9] is proposed as a multi-scan method offering improved performance in clutter over the conventional single-scan probabilistic data association filter (PDAF) [6]. While in [9], VDA has been extended to multi-target tracking scenarios, in this paper VDA is proposed for tracking in an uncertain multipath channel.

The organization of this paper is as follows. Section 2 introduces an indoor multipath propagation model based on specular reflection and the method of image. Section 3 formulates the recursive state estimation and data association problem for multi-mode geolocation. Section 4 derives multi-mode VDA for single target geolocation and an illustrative simulation is presented in Section 5. Finally, conclusions and future work are discussed in Section 6.

2 Multipath Propagation Modeling

In the following sections, locating a single RF emitter in an uncertain indoor environment is considered to illustrate the concept of simultaneous target and multipath channel estimation. We model the indoor multipath measurements based on geometric optics, and the method of images is employed for two-dimensional scenarios. To simplify the propagation model and reduce computational complexity, all reflections (i.e., from walls, floors and partitions) are assumed to be specular. Through-wall propagation and the effects of diffraction and scattering are ignored under the assumption that these arrivals are much weaker than the specular components.

As discussed in the case of SLAM [4], several types of indoor features may be considered: plane surfaces, convex and concave edges. Plane surfaces produce specular images which exhibit “inverse” trajectories with respect to
direct path, while convex and concave edges are often considered as point reflectors. Arc reflectors are also possible but uncommon, and thus are not discussed here. For localization purposes, multipath returns from plane surfaces are most desirable due to their high correlations with the direct path.

Consider a one-way propagation case to track a moving emitter \( s(t) = [x_e(t), y_e(t)]^T \) with x-y locations based on angle-of-arrival (AOA) and time-of-arrival (TOA) measurements at known receiver \( r \). Figure 1 illustrates an example with first-order reflection, where the signal from source \( s(t) \) intersects a plane \( P_k \) and bounces from the specular point to the receiver \( r \). Denoting the azimuth AOA and TOA measurements for this path as \( \theta(t, k) \) and \( r(t, k) \) respectively, an image or virtual source \( s'(t, k) \) with respect to \( P_k \) is given by:

\[
s'(t, k) = r(t, k) \begin{bmatrix} \cos \theta(t, k) \\ \sin \theta(t, k) \end{bmatrix} + r
\]

(1)

Denoting plane \( P_k \) as \( \cos \psi_k x + \sin \psi_k y + \rho_k = 0 \), the relationship between \( s'(t, k) \) and \( s(t) \) can be written as:

\[
s'(t, k) = E_k s(t) + b_k
\]

(2)

where \( E_k = \begin{bmatrix} -\cos 2\psi_k & -\sin 2\psi_k \\ -\sin 2\psi_k & \cos 2\psi_k \end{bmatrix} \)

\( b_k = -2\rho_k \begin{bmatrix} \cos \psi_k \\ \sin \psi_k \end{bmatrix} \)

Therefore, a NLOS path \( k \) can be defined by the parameter vector \( \theta_k(t) = [\psi_k, \rho_k]^T \), which is observable given a \( s'(t, k) \) and \( s(t) \) pair for a single dwell. An interesting case is jointly estimating true source position and path parameters for track initialization if the direct path is initially not present, which requires at least 3 indirect paths and 2 dwells. With noisy measurements, this joint estimation problem can be solved by a nonlinear least square method associated with a nearest-neighbor filter or a joint probabilistic data association filter (JPDA) [6] for track denoising. Furthermore, the single bounce case can be directly extended to multiple bounce cases by cascading equation (2) which is discussed in [5].

3 State-Space Formulation

At discrete time \( t \), suppose there are \( m(t) \) measurements observed and \( M(t) \) possible propagation modes (i.e., \( M(t) \) paths, both direct and multipath from reflected planes), characterized by the mode parameter vector \( \theta_k(t), k = 1, ..., M(t) \). Define the mode set \( \Omega(t) \) as all the \( M(t) \) modes with index \( [1, ..., M(t)] \), and the measurement set \( Z(t) \), with index \( [1, ..., m(t)] \). For each mode \( k \), define \( m(t) \) + 2 possible states: mode not present, mode present but missed and mode present referring to \( m(t) \)th measurement, as shown in Figure 2.

\[
\begin{array}{c}
\text{mode } k \\
\text{detected} \\
\text{not present} \\
\text{missed} \\
\text{refer to } m(t)\text{th measurement}
\end{array}
\]

Figure 2: Hypothesis tree for \( k \)th mode at time \( t \)

The data association problem is to then find the optimal correspondence between \( Z(t) \) and \( \Omega(t) \), denoted as \( \Phi(t) \), i.e.,

\[
\Phi(t): \{1, ..., M(t)\} \rightarrow \{-1,0,...,m(t)\}
\]

where

-1: mode not present
0: mode missed

Therefore, the association \( \Phi(t) = [\phi_1(t), ..., \phi_M(t)] \) represents a \( 1 \times M(t) \) vector whose elements can range from -1 to \( m(t) \), with non-repeated 1 to \( m(t) \) according to the assumption that each measurement can be associated with only one mode at one time. The measurements without assignments originate from false alarms or new modes. It can be shown that the total number of data association hypotheses at time \( t \) is given by:

\[
N_\Phi(t) = \sum_{i=0}^{\min\{m(t),M(t)\}} \binom{M(t)}{i} \frac{m(t)!}{(m(t)-i)!} \sum_{j=0}^{M(t)-i} \binom{M(t)-i}{j}
\]

(3)

Let the system dynamic model of state and measurement equations be defined as:

\[
x(t+1) = F(t)x(t) + w(t), \quad w(t) \sim \mathcal{N}(0,Q(t))
\]

(4.1)

\[
z_{\phi_k(t)}(t) = h_k(x(t)) + v_k(t), \quad v_k(t) \sim \mathcal{N}(0,R_k(t))
\]

(4.2)

where the state vector \( x(t) \) includes both target parameters (e.g., geographical coordinates and vector velocity) and mode parameters for all the \( M(t) \) modes, i.e.,

\[
x(t) = [x(t), \dot{x}(t), y(t), \dot{y}(t), \theta_1(t)^T, ..., \theta_{M(t)}(t)^T]^T
\]
where $h_k(\cdot)$ is the observation function for $k^{th}$ mode with respect to the TOA and AOA measurements as defined in (1) and (2), and the state transition matrix $F(t)$ defining target and reflective surface dynamics is given by:

$$F = \begin{bmatrix}
1 & T & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & T \\
0 & 0 & 0 & 1
\end{bmatrix}$$

where $T$ is the inter-dwell sampling interval. Note that since the reflected planes are assumed to be static, the state noise is zero for the mode parameters. The observation noise covariance matrix $R(t)$ is given by

$$R(t) = \begin{bmatrix}
\sigma_r^2 & 0 \\
0 & \sigma_\theta^2
\end{bmatrix}$$

where $\sigma_r^2$ and $\sigma_\theta^2$ are functions of signal time-bandwidth product and SNR.

Finally, $\phi_k(t)$ denotes the measurement index corresponding to $k^{th}$ mode under the specified hypothesis. If $\phi_k(t) = -1$ or 0, no measurement is associated with mode $k$. For the measurement index excluded by $\phi_k(t)$, the corresponded measurement may refer to a false alarm or new mode.

### 4 Multi-Mode Viterbi Data Association

Unlike conventional discrete-state hidden Markov modeling approaches, where the nodes in a trellis represent the discretized target states, or previous single mode single target Viterbi data association algorithms [7, 9], we define the nodes in the trellis as each data association hypothesis $\Phi(t)$, from the hypothesis set $\Phi(t) = \{\Phi_1(t)\}_{t = 1, N\Phi(t)}$, as shown in Figure 3.

$$d_i(t) = \min_{\Phi_k(t)} - \ln p(Z^t, \Phi_k(t) | X^t)$$

where $\Phi_k^{t*}$ is the most likely hypotheses sequence terminating at node $\Phi_k(t)$, with the cumulative joint log likelihood $d_i(t)$. It can be seen that, as the number of dwells increases, the computational complexity will exponentially increase. This complexity can be reduced substantial by dynamic programming for a model where the joint likelihood at node $\Phi_k(t)$ can be decomposed as

$$p(Z^t, \Phi_k(t), X^t) = p(Z(t), Z^{t-1}, \Phi_k(t), X^t)$$

$$= p(Z(t), \Phi_k(t) | X^t, \Phi^{t-1}) \times p(Z^{t-1}, \Phi^{t-1} | X^{t-1})$$

Since the true target and parameter state sequence $X^t$ is unknown, in VDA the estimated states $\hat{X}^t$, which depends on both $\Phi^{t-1}$ and $Z^{t-1}$, are used to approximate the optimal solution. We define the transition log likelihood

$$a_{ij}(t) \triangleq - \ln p(Z(t), \Phi_i(t) | X^t, \Phi^{t-1}_j)$$

$$\equiv - \ln p(Z(t), \Phi_i(t) | Z^{t-1}, \Phi^{t-1}_j)$$

$$= - \ln p(Z(t), \Phi_i(t) | x_i(t | t-1), \Phi_{j}(t-1))$$

(8)

where $\Phi^{t-1}_j$ is the most likely hypotheses sequence terminating at node $\Phi_j(t-1)$, and $x_i(k | k-1)$ is the state prediction based on the node $\Phi_i(t-1)$. Thereby, the optimization problem can be rewritten as:

$$j^*_i = \arg\min_{\Phi^{t-1}_j} a_{ij}(t) + d_j(t-1)$$

(9)

$$d_i(t) = d_{i^*_i}(t) = \min_{\Phi^{t-1}_j} a_{ij}(t) + d_j(t-1)$$

(10)

Thus, the state vector updates at time $t$ associated with node $\Phi_i(t)$ can be calculated as:

$$x_i(t | t) = x_{i^*_i}(t | t)$$

(11)

Note that VDA significantly reduces the computational loads by only keeping the prediction from $\Phi_{j^*_i}(t-1)$ for node $\Phi_i(t)$.

### 4.1 Gating

We consider a gating process as described in [9] to reduce the increasing computational cost due to large $M(t)$ and $m(t)$. At time $t$, a measurement $Z(t)$ is defined as a valid measurement for $k^{th}$ mode from node $\Phi_j(t-1)$ if it falls in the $|Z(t)|$ dimensional validation region $Y_k(t)$, i.e.,

$$Y_k(t) = \{z(t) : |z(t) - \hat{z}_{k,j}(t | t-1)|^2 S_{k,j}(t)^{-1} \leq \gamma \}$$

(12)
where \( \mathbf{z}_{k,j}(t|t-1) \) and \( \mathbf{s}_{k,j} \) are predicted measurement and innovation covariance matrix from node \( \Phi_j(t-1) \) for mode \( k \), and \( y \) is defined by a chi-squared distribution with \(|\mathbf{z}(t)|\) degrees of freedom associated with the gate probability \( P_G \), which is the probability that the validation region \( \mathbf{y}_k(t) \) contains true measurement.

### 4.2 Evaluation of Transition Likelihoods

The mode presence dynamics is modeled as a two-state Markov chain with transition matrix:

\[
\begin{bmatrix}
    \alpha_k & 1 - \alpha_k \\
    1 - \beta_k & \beta_k
\end{bmatrix}
\]  

(13)

where

\[
\alpha_k = \Pr\{\text{present at } t | \text{present at } t-1\}
\]

\[
\beta_k = \Pr\{\text{not present at } t | \text{not present at } t-1\}
\]

The state dynamics of mode \( k \), according to Section 2 is then illustrated in Figure 4, where \( P_D \) and \( P_G \) denotes probability of detection (functions of SNR and false alarm rate) and gate probability introduced in section 4.1.

![Figure 4: Three state mode state Markov chain](image)

Assuming all the mode states are independent at time \( t \), under hypotheses \( \Phi_i(t) \) and \( \Phi_j(t-1) \), define the mode set \( \Omega(t) \) which is given by:

\[
\Omega(t) = \Omega_1 \cup \Omega_2 \cup \Omega_3 \cup \Omega_4 \cup \Omega_5 \cup \Omega_6
\]

(14)

where

\( \Omega_1 \): modes detected at \( t \) and present at \( t-1 \)

\( \Omega_2 \): modes detected at \( t \) and not present at \( t-1 \)

\( \Omega_3 \): modes not present at both \( t \) and \( t-1 \)

\( \Omega_4 \): modes not present at \( t \) and present at \( t-1 \)

\( \Omega_5 \): modes missed at \( t \) and present at \( t-1 \)

\( \Omega_6 \): modes missed at \( t \) and not present at \( t-1 \)

The mode state corresponding to the measurement set \( Z(t) \) is then:

\[
Z(t) = Z_t \cup Z_f
\]

(15)

where

\( Z_t \): target oriented measurements (\(|Z_t| = |\Omega_1 \cup \Omega_2|\))

\( Z_f \): measurements due to false alarm or new modes

For simplification, the hypotheses index \( i, j \) and time index \( t \) are dropped. Therefore, the transition log likelihood \( a_{ij}(t) \) in (8) can be written as:

\[
a_{ij}(t) = - \ln p(\mathbf{x}_j(t)|t-1, \Phi_i(t))
\]

\[
\frac{(P_D P_G)}{(1 - P_D P_G)} \prod_{k_1 \in \Omega_1 \cup \Omega_2} N(z_{\Phi_i k_1}, \mathbf{z}_{\Phi_i k_1}, \mathbf{s}_{\Phi_i k_1})
\]

\[
= - \ln \left( \prod_{k_2 \in \Omega_3 \cup \Omega_4} (1 - \beta_{k_2}) \prod_{k_3 \in \Omega_5 \cup \Omega_6} \alpha_{k_3} \right)
\]

(16)

where the false alarm spatial density is assumed to be uniformly distributed over the total surveillance volume \( V \). In (16), the statistical correlations between measurements are ignored, which saves computational resource with fewer penalties. However, these correlations have to be addressed when updating the state with the extended Kalman filter.

### 4.3 Pruning of Hypotheses

VDA facilitates optimal depth control pruning using the recursion in (7) and gating process is effective in width control by eliminating hypotheses corresponding to invalided measurements. However, as \( M(t) \) and \( m(t) \) increase, additional width pruning scheme may be utilized to eliminate low-score hypotheses.

![Figure 5: Simplified mode Markov chain](image)

The trellis pruning is based on thresholding the joint likelihood \( p(Z; \Phi_j | X_f) \) and validating the predicted modes states. For example, a hypothesis that none of the modes present at \( t \) followed by a hypothesis that all the
modes present at time $t - 1$ is quite unlikely in presence of large number of modes.

The hypothesis trellis can be further simplified by combing $-1$ and 0 states (i.e., the mode does not exist and mode miss detected). For example, the three-state Markov chain in Figure 5 can be simplified into two states: mode associates with a measurement or not, as shown in Figure 7. Therefore, the mode set $\Omega_3$ and $\Omega_4$ can be clustered with $\Omega_5$ and $\Omega_6$, and parameterized by the new transition probabilities $q_k$ and $p_k$.

### 4.4 Birth of New Modes

In this paper, we consider a relative simple mechanism with respect to the birth of new modes. It is not possible to distinguish a measurement from a new mode versus a false alarm using only a single dwell, so depending on the environment and the false alarm rate, two approaches may be used [11]. At low false alarm rate, a measurement associated with no existing mode is treated as a new mode until being proved as a false alarm in the next few dwells. Alternatively, at high false alarm rate, a measurement associated with no mode can be treated as a false alarm until being proved as a new mode in the next few dwells. Since we mainly consider high clutter indoor or urban environments, the second approach is chosen for detecting new modes.

### 5 Simulations

To evaluate the performance of the proposed algorithm, consider a two-dimensional moving emitter tracking problem in an uncertain indoor RF multipath environment as shown in Figure 6. A receiver array is located at the origin (the receiver is assumed to be perfect synchronized with the emitter, hence the absolute TOA is available) and the multipath propagation is assumed to be single bounce specular reflection as introduced in section 2. The emitter moves clockwise from the upper-right corner of the room with number of modes $M(t)$ increasing from one (direct path) to five (direct path and four multipath). The gray area in Figure 6 denotes the NLOS area due to the two obstructions, and the direct path is occasionally unobservable along the emitter trajectory.

The observation noise variance for TOA and AOA are based on the Cramer-Rao bound (CRB) model as [12, 13]:

$$
\sigma_r = \frac{c}{BW \sqrt{SNR}}, \quad \sigma_\theta = \frac{2}{N \sqrt{SNR}}
$$

where $c$ is the speed of light, $BW$ is the RF bandwidth (assumed to be 600MHz) and $N$ is the number of array elements (assumed to be 16). The SNR of direct path is assumed as 20 dB with $P_D = 0.9$, while the multipath signal is set as 14 dB with $P_D = 0.6$. The other parameter values assumed for the simulation $P_\gamma = 0.999$ with $\gamma = 13.82$ and the false alarm rate is assumed as $\lambda = 10^{-2}$ per unit volume. The total number of dwells used is 120 with inter-dwell sampling interval $T = 1$ s. The mode presence transition probability $\alpha_k$ and $\beta_k$ are both assumed to be 0.95 for all modes. The simplified two-state mode Markov chain introduced in Section 4.3 is used, obtained by combining -1 and 0 states into a single state 0, when mode $k$ has no measurement associated with. To simplify the data association problem, we also assume the true state noise covariance matrix $Q(t)$ of the emitter is known exactly for each scan, in order to deal with the maneuvering dynamics of the source.

In Figure 7, the estimated trajectory of the emitter is shown along with the ground truth of the most likely hypotheses sequence terminated at the last scan. The corresponding localization RMS errors for x-y positions are illustrated in Figure 8. Even though the direct path is occasionally undetectable due to miss detections or obstructions, the receiver array is still able to produce very small tracking errors by navigating off the indirect path from the walls. Furthermore, utilizing multipath signals is able to reduce the RMS errors and track uncertainty compared with using direct path alone.

![Figure 6: Example of simulation environment (the gray area refers to NLOS region)](image)

![Figure 7: Estimated and true emitter track](image)
Figure 8: RMS errors for emitter position estimates of the optimal hypothesis sequence

Figure 9: RMS errors for multipath parameter estimates (a) $\psi$ and (b) $\rho$ of the optimal hypothesis sequence

Figure 10: Mode state estimates (solid red line) vs. ground truth (blue cross)
Figure 9 shows the multipath parameter (i.e., $\psi$ and $\rho$ in equation (2) introduced in section 2) estimation performance for the four multipath modes of the most likely hypotheses sequence, where both $\psi$ and $\rho$ converge to the true value as time going on. The estimated and the true mode states are shown in Figure 10, where very few association errors can be observed.

6 Conclusion

This paper proposes a novel algorithm for the problem of joint estimation of target location and multipath parameters. A simplified ray-tracing based multipath measurement modeling is introduced to illustrate the proposed algorithm. The data association problem is solved using a multi-mode Viterbi data association algorithm. An illustrative simulation of an indoor single target tracking problem is presented which demonstrates that both multipath mode parameters and target positions can be jointly estimated even when direct path propagation is not always present. Future work includes the derivation of improved trellis pruning schemes which would facilitate computationally-efficient solution of more complicated multi-target tracking problems.

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References


