An extended target tracking method with random finite set observations

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Abstract—A target is denoted extended when the target extent is larger than the sensor resolution. A tracking algorithm should be capable of estimating the target extent in addition to the state of the centroid. This paper addresses the problem of tracking an extended target with unknown parameter about the target extent. The extended target is regarded as a spatial distribution model, and the target extent is considered as a mixture of multiple probability distributions in this paper. The EM algorithm is utilized to estimate the unknown parameters about target extent, and also a particle implementation of the presented method is given. Simulation results validate the effectiveness of the presented method.

Keywords: Extended target, target extent, random finite sets.

1 Introduction

In general tracking applications, the target to be observed is usually considered as a point source object. That is, the target extent can be neglected compared with the sensor resolution. With the increasing sensor resolution, the sensor can detect more than one measurement from the same target. What one should concern includes both the state of the centroid and the target extent.

For an extended target tracking problem, there exist a variety of approaches for incorporating the target extent into target tracking process [1-13]. A natural way is to regard the extended target as a set of rigid points, each of which can be a source of sensor measurements. In this way, one needs to construct the explicit assignments between the measurements and the sources. However, if the number of measurement source is large, such a method based on data association is most challenging and also unnecessary. Another alternative approach is to model the extended target as an intensity distribution rather than a set of points. Each target-related measurement is an independent sample from the spatial distribution. Such an approach makes it possible to compute the likelihood function without construct the explicit association hypothesis between the measurements and the sources.

The target extent is closely related to the target shape. One typically approximates the shape by means of a basic geometric shape such as a line, a rectangle or an ellipse [2, 6, 11]. However, for the target with slightly complex shape such as an airplane or a ship, modeling the target extent with a basic geometric shape is not enough.

Generally, the motion pf a point target is modeled by its centroid’s dynamic model. For an extended target, an intuitive idea is to model its centroid as done in point target case, and to treat its extent as randomly distributed samples in the target plane, which is regarded as a finite mixture model so as to incorporate the complex shape of the extended target. And also there are some unknown parameters about the finite mixture model.

The random finite sets (RFSs) theory [14-15] provides a promising tool to implement a mathematically consistent generalization of Bayesian recursion formula, from single-target single-measurement case to set-valued case. In this paper, what we do firstly is to estimate those parameters about the target extent based on a sequence of measurements, and then update the centroid state under the RFS theory frame.

This paper is organized as follows. A problem formulation of extended target tracking using random finite set theory is formulated in section2. In Section 3 we introduced the EM algorithm for extended target tracking with unknown parameters. A particle implementation of the presented method is given in section 4. Simulation results are presented in section 5. Conclusions and discussions are given in section 6.

2 Problem formulation

In extended target tracking application, the sensor can detect more than one measurement originating from the same target. In this paper, we try to address the extended target tracking problem from the view of general Bayesian recursion.

2.1 General Bayesian recursion

Let \( x_k \) and \( z_{1:k} \) denote the target state and measurement at time \( k \). The cumulative measurement set up to time \( k \) is denoted by \( z_{1:k} \). The target state \( x_k \) follows a first-order
Markov transition process according to a transition density \( f_{k\mid k-1}(x_k | x_{k-1}) \). The measurement likelihood is given by \( g_k(z_k | x_k) \). According to the classical Bayesian recursion, the posterior density \( P_{\mid k-1}(x | z_{k-1}) \) about target state is propagated as follows:

\[
P_{\mid k-1}(x_k | z_{k-1}) = \int f_{k\mid k-1}(x_k | x) P_{\mid k-1}(x | z_{k-1}) dx
\]

\[
P_{\mid k}(x_k | z_{k-1}) = \frac{g_k(z_k | x_k) P_{\mid k-1}(x_k | z_{k-1})}{\int g_k(z_k | x_k) P_{\mid k-1}(x_k | z_{k-1}) dx}
\]

2.2 RFS measurement model for multiple measurements case

In many cases such as extended target tracking and multi-path reflections, one target can produce multiple measurements. The general Bayesian recursion is extended to accommodate multiple measurements cases in [14]. The key step is to derive a consistent likelihood function based on random finite set theory. The related results are given as follows.

In a multiple measurement case, the measurement is a set-valued variable. Let \( \mathcal{Z} \) denote the measurement space, \( m_k \) be the number of measurements at time \( k \). Then, all the measurements at time \( k \) are \( z_{k,i} \in \mathcal{Z} \) \((i = 1, \ldots, m_k)\). The measurement set at time \( k \) can be described as the random finite set variables:

\[
Z_k = \{z_{k,1}, z_{k,2}, \ldots, z_{k,m_k}\}, Z_k \in \mathcal{F} (\mathcal{Z})
\]

where \( \mathcal{F} (\mathcal{Z}) \) represent the respective collections of all finite set observations generated by \( \mathcal{Z} \).

According to the measurement origin, the total measurement RFS at time \( k \) can be classified into the following three parts:

\( \Theta_k(x_k) \) : the RFS of primary target-generated measurement which corresponds with the one from the target centroid.

\[
\Theta_k(x_k) = \begin{cases} 
\emptyset & \text{with probability } 1 - P_d \\
\{z_k^*\} & \text{with probability density } P_d g_k(z_k^* | x_k)
\end{cases}
\]

where \( g_k(\cdot | \cdot) \) and \( P_d \) are the likelihood and the probability of detection for the primary measurement, respectively.

\( W_k \) : the clutter RFS which is modelled as a Poisson RFS with the intensity \( v_{W_k}(z_k) \).

\( E_k(x_k) \) : the RFS of extraneous target-generated measurement corresponding with the measurements from the target extent, which is also modelled as a Poisson RFS with the intensity \( v_{E_k}(z_k | x_k) \).

And also we assume that these three RFS are independent. By integrating two RFS \( W_k \) and \( E_k(x_k) \) together, we have

\[
K_k(x_k) = E_k(x_k) \cup W_k
\]

The intensity of RFS \( K_k(x_k) \):

\[
v_{K_k}(z_k | x_k) = v_{E_k}(z_k | x_k) + v_{W_k}(z_k)
\]

The cardinality distribution \( \rho_{K_k}(n | x_k) \) of \( K_k(x_k) \) is Poisson with mean \( \int v_{K_k}(z_k | x_k) dz_k \).

For multiple measurement cases, the key step is to derive the probability density \( \eta_k(Z_k | x_k) \) that the state \( x_k \) produce the measurement set \( Z_k \), which corresponds with the single measurement likelihood \( g_k(z_k | x_k) \) in formula (2). The probability density \( \eta_k(Z_k | x_k) \) is given in [14] by

\[
\eta_k(Z_k | x_k) = [1 - P] \rho_{K_k}(|Z_k| | x_k) |Z_k| \prod_{z_k \in Z_k} c_k(z_k | x_k)
\]

\[
+ P P_d \rho_{E_k, d}(|Z_k| - 1 | x_k) |Z_k| - 1 \prod_{z_k^* \in Z_k} g_k(z_k^* | x_k) \prod_{z_k \in Z_k} c_k(z_k | x_k)
\]

where \( c_k \) means the probability density of measurement \( z_k \) in RFS \( K_k(x_k) \) given the system state \( x_k \)

\[
c_k(z_k | x_k) = \frac{v_{K_k}(z_k | x_k)}{\int v_{K_k}(z | x_k) dz}
\]

3 Extended target tracking with random finite set observations

3.1 The RFS Bayesian recursion for single extended target.

As described above, if all the parameters in the intensity \( v_{K_k}(z_k | x_k) \) are known, then the likelihood function \( \eta_k(Z_k | x_k) \) can be obtained directly by formula (5). Unfortunately, the measurements from the target extent are usually produced by an unknown distribution. In this case, we need to approximate the unknown distribution firstly. In this paper, we assume that the extraneous target-generated measurements originate from the mixture of multiple probability density function, and each component \( E_{k,q}(x_k) (q = 1, \ldots, M) \) is a Poisson RFS with intensity \( v_{E_{k,q}}(z_k | x_k) \), with the mean \( \lambda_{z_k,q} \) for the cardinality distribution, and \( M \) is the number of mixture components.

The whole RFS for all extraneous target-generated measurements are modelled as the union of multiple Poisson RFS. To unify the notations, we define
\[ \Omega_{k,i}(x_i) = \begin{cases} \emptyset(x_i) & i = 1 \\ W_k & i = 2 \\ E_{k,j} & 2 \leq i \leq M + 2 \end{cases} \]

By using of standard measurement theoretic probability, we can derive the probability density that the state \( x_k \) produces the measurement set \( Z_k \) as done in [14]. For any Borel subset \( S \subseteq \mathcal{F}(\mathcal{L}) \), decomposing \( S \) into \( S = \bigcup_{r=0}^{\infty} S_r \), where \( S_r \) is the subset of \( S \) with the length \( r \).

\[
P_k(S \mid x_k) = \sum_{r=0}^{\infty} P_k(Z_k \in S \mid x_k) = \sum_{r=0}^{\infty} P_k(Z_k \in S \mid z_k) \]

Define the events set \( \{\xi_{k,r}\}_{k=1}^{N_r}, \) \( N_r \) is the number of all feasible events. \( \xi_{k,r} = \{\xi_{k,1}, \ldots, \xi_{k,r}\} \), \( n_{k,r} \) is the component index which indicates that measurement \( z_{k,r} \) comes from component \( i_{k,r} \) in event \( \xi_{k,r} \).

\[
f_{k,j}^{(i)}(z_{k,j} \mid x_k) = \begin{cases} 1 & \text{from the centroid} \\ 2 & \text{from clutter} \\ s(s > 2) & \text{from taget extent} \end{cases}
\]

According to the total probability formula, we have

\[
P_k(S \mid x_k) = \sum_{i=1}^{N_r} P(S \in \xi_{k,r} \mid x_k) P(S \in \xi_{k,r} \mid x_k)
\]

We introduce \( n_{k,r} \) (2 \( \leq s \leq M + 2 \)) as the measurement count from component \( s \) in event \( \xi_{k,r} \). By combining the detection probability \( P(d) \), the cardinality distribution for each Poisson RFS, and possible permutations of measurement set, the probability \( P(S \in \xi_{k,r} \mid x_k) \) is given by

\[
P(S \in \xi_{k,r} \mid x_k) = \left(1 - P(d)^{M+2}\right)^{\delta_k} \prod_{s=1}^{M+2} \rho_{s,k}(n_{s,k} \mid x_k)
\]

where \( \delta_k \) is a binary variable indicating whether the centroid of extended target is detected or not. If there exists a measurement coming from the centroid in event \( n \), then \( \delta_k \) is set to be 1; otherwise, it is set to be zero. We denote \( \rho_{s,k} \) be the cardinality distribution of the RFS \( \Omega_{k,s}(x_k) \) (2 \( \leq s \leq M + 2 \)).

The density \( P_k(Z_k \in S \mid \xi_{k,r}^{(i)} \mid x_k) \) can be obtained by appropriately integrating over \( S_r \), the density \( \eta_k(Z_k \mid x_k) \), where

\[
\eta_k(Z_k \mid x_k) = \prod_{j=1}^{M+2} f_{k,j}^{(i)}(z_{k,j} \mid x_k)
\]

\[
P_k(S \mid x_k) = \sum_{r=0}^{\infty} P_k(Z_k \in S \mid x_k) = \sum_{r=0}^{\infty} P_k(Z_k \in S \mid z_k)
\]

where \( \chi \) is the \( r \) th product Lebesgue measure on \( \mathcal{L}' \). We have

\[
P_k(S \mid x_k) = \sum_{r=0}^{\infty} P_k(Z_k \in S \mid z_k)
\]

3.2EM algorithm for extended target tracking with unknown parameters

When there are some unknown parameters \( \theta = \{\theta_i\}_{i=1}^{M+2} \) about the intensity function \( \lambda_{k,j} \), the likelihood \( \eta_k(Z_k \mid x_k) \) will be replaced by its maximum likelihood function \( \hat{\eta}_k(Z_k \mid x_k) \).

\[
\hat{\eta}_k(Z_k \mid x_k) = \max_{\theta} \{\eta_k(Z_k \mid x_k, \theta)\}
\]

In this section, EM algorithm is used to produce the estimates for these unknown parameters. The EM
algorithm is an iterative algorithm for parameter estimation when the data set is incomplete. It can be applied for not only ML estimation, but also MAP estimation. It carries out two steps alternately (E step and M step) until the algorithm converges.

**E step:** Find the expected value of the complete-data log-likelihood with respect to the miss data \( l_k = \{l_{k,j}\}_{j=1}^m \) given the observed data and the current parameter estimates. That is

\[
Q(\theta, \hat{\theta}^g) = E_{\theta^g}\left\{ \log G_k(Z_k, l_k | x_k, \theta) \right\}
\]

The first item in equation (17) can be computed as

\[
= \sum_{l_k} \log G_k(Z_k, l_k | x_k, \theta) p(l_k | x_k, Z_k, \hat{\theta}^g)
\]

\[
= \sum_{l_{k,j=1}}^{M+2} \sum_{l_{k,j=1}}^{m} \left( \sum_{j=1}^{m} \frac{m}{\pi} \right) \log p(l_k | x_k, \theta))p(l_k | x_k, Z_k, \hat{\theta}^g)
\]

\[
= \sum_{l_{k,j=1}}^{M+2} \sum_{l_{k,j=1}}^{m} \log p(l_k | x_k, \theta))p(l_k | x_k, Z_k, \hat{\theta}^g)
\]

\[
= \sum_{l_{k,j=1}}^{M+2} \sum_{l_{k,j=1}}^{m} \log (f_{l_{k,j}}(z_{k,j} | x_k, \theta))p(l_k | x_k, Z_k, \hat{\theta}^g)
\]

\[
= \sum_{l_{k,j=1}}^{M+2} \sum_{l_{k,j=1}}^{m} \log (f_{l_{k,j}}(z_{k,j} | x_k, \theta))p(l_k | x_k, Z_k, \hat{\theta}^g)
\]

\[
= \sum_{l_{k,j=1}}^{M+2} \sum_{l_{k,j=1}}^{m} \log (f_{l_{k,j}}(z_{k,j} | x_k, \theta))p(s | x_k, z_{k,j}, \hat{\theta}^g)
\]

The second item in equation (17) is

\[
P(l_k | x_k, \theta)
\]

\[
P_D \delta (1 - P_D) \prod_{s=2}^{M} e^{-\lambda_{\Omega,s}} \frac{n_s!}{m_n!} C_{n_m} C_{n_{m-1}} \ldots C_{n_{m-n_m+2}}
\]

\[
= \sum_{l_{k,j=1}}^{M+2} \sum_{l_{k,j=1}}^{m} \log (f_{l_{k,j}}(z_{k,j} | x_k, \theta))p(s | x_k, z_{k,j}, \hat{\theta}^g)
\]

where

\[\hat{\lambda}_{\Omega,s}\]

is the mean of the cardinality distribution for

Poisson RFS \( \Omega_{k,s} \) \((2 \leq s \leq M + 2)\)

The updated parameter can be obtained by taking the derivative with respect to the parameters \( \theta \) (for \( 2 \leq s \leq M + 2 \)) and set the derivative to be zero.

### 4 Particle filter implementation

In this section, we give a particle implementation of the above method.
Step 1: Given a group of particles \( \{ x^{(i)}_{k-1}, \omega^{(i)}_{k-1} \} \) which represent the centroid state of the extended target at time \( k-1 \).

Step 2: Estimate the unknown parameters \( \vartheta \) about the target extent by using of the measurement set \( Z_k \) as described in section 3.2. The target state \( x^{(i)}_k \) is approximated by the predictive state of the centroid.

Step 3: Sample \( x^{(i)}_k \sim q(. | x^{(i)}_{k-1}, Z_k) \), where \( q(. | .) \) is the proposal distribution.

Step 4: Compute the weights
\[
\tilde{\omega}^{(i)}_k = \omega^{(i)}_{k-1} \frac{\eta_k(Z_k | x^{(i)}_k) f_{x | \omega}^{(i)}(x^{(i)}_k | x^{(i)}_{k-1})}{q_k(x^{(i)}_k | x^{(i)}_{k-1}, Z_k)}
\]
The normalized weights is
\[
\omega^{(i)}_k = \frac{\tilde{\omega}^{(i)}_k}{\sum_i \tilde{\omega}^{(i)}_k}
\]

Step 5: The centroid state is approximated as
\[
\hat{x}_k \approx \sum_i \omega^{(i)}_k x^{(i)}_k
\]

5. Simulation results

The following example is provided to validate the effectiveness of the presented extended target tracking method.

**Dynamic model:**
The motion of the target centroid follows the following dynamic equation:
\[
x(k+1) = \Phi x(k) + \Gamma v(k)
\]
where \( x(k) = [x(k), \dot{x}(k), y(k), \dot{y}(k)]^T \), which means the position and velocity of the target centroid at x-y plane, \( \Phi = diag\{\Phi_1, \Phi_2\} \), \( \Gamma = diag\{\Gamma_1, \Gamma_2\} \), \( \Phi_1 = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \), \( \Gamma_1 = \begin{bmatrix} T^2 / 2 \\ T \end{bmatrix} \), the sample interval \( T \) is 1s. The initial state of the target centroid is \( x_0 = [200 1 200 2]^T \), and the covariance matrix of the process noise is designed as \( Q = diag\{9, 4\} \). We assume that spatial distribution of target extent is the mixture of two Gauss distribution, with the mean at the target centroid and the covariance \( \Sigma_{k,1} = diag\{30, 0.2, 1, 0.2\} \), \( \Sigma_{k,2} = diag\{4, 0.1, 40, 0.2\} \).

**Measurement model:**
We assume only the position information of the measurement source is detected, so the measurement equation for primary target-generated measurements is
\[
z(k) = Hx(k) + w(k)
\]
where the measurement matrix is \( H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \).

The covariance matrix of the measurement noise is designed as \( R = 2I \), and \( I \) is an \( 2 \times 2 \) identity matrix.

For the extraneous target-generated measurements, they are modeled as the mixture of two Poisson RFS with intensity \( \lambda_{w_1}(z_k) = \sum_i \lambda_{w_i} N(z_k; H \hat{x}_k, \Psi_{w_i}) \),
\[
\lambda_{w_{1,2}} = 50, \lambda_{w_{1,2}} = 60, \Psi_{w_{1,2}} = H\Sigma_{w_{1,2}}H^T + R.
\]
Clutter is also modeled as a Poisson RFS with intensity \( \lambda_w(z_k) = \lambda_w u(z_k) \), where \( u(z_k) \) is the uniform distribution over the observation region. \( \lambda_w \) is the expected number of clutter, which is given an average \( \lambda_w = 20 \).

To implement the particle filter, 500 particles are used at each time step, and the transition is used to be the proposal. The measurements are supported by 100 MC runs performed on the same target trajectory but with independently generated measurements for each trial. Fig 1 shows the real and estimated position for the target centroid in one trial by using of Probability Data Association method (PDA) and the presented method, respectively. The RMSE curve of the target centroid is illustrated in Fig. 2.

**Fig. 1** the real and estimated position for the centroid

**Fig. 2** RMSE of the target centroid

Fig. 3 shows the trajectory of the target centroid, the measurements from the target extent based on the mixture distribution and the estimated target extent (illustrated by two joint ellipsoids) for several snapshots in one trial. In
Fig.4, the estimated and real target extents are shown at scan $t = 30$. 

6 Conclusions and discussions

In this paper, an effective approach is developed to deal with the extended target tracking problem with unknown parameters about target extent. The presented approach approximates the target extent by using of a finite mixture model, rather than a simple geometric shape. Simulation results show its effectiveness to deal with extended target tracking problem. However, when the extended target makes the fast rotation movement and the sensor only receives small amount of measurements at each scan, the presented method doesn’t work well and multi-scan information is needed to achieve good performance. More effective method to deal with the fast rotation movement will be done in further research.

Acknowledgement

The work is sponsored by the national key fundamental research & development programs (973) of P.R. China (2007CB311006) and National Natural Science Foundation of China (61004087/F030119).

Reference


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